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Mathematics Teacher Preparation Standards in a Research-based Mathematics Course for Prospective Secondary Mathematics Teachers: Exploring Student Experiences and Curriculum Writers' Perspectives

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Abstract

Recommendations for preparing teachers of mathematics have been published by multiple professional organizations in the United States; however, there is limited research on whether student course perceptions affirm the instructor-identified alignment of mathematics content courses for prospective secondary mathematics teachers (PSMTs) to these recommendations. We investigate the alignment of research-based tasks and explorations used in a content course for PSMTs to the Association of Mathematics Teacher Educators Standards for the Preparation of Teachers of Mathematics and recommendations in the Mathematical Education of Teachers II. Analyzing end-of-course interviews with 11 students enrolled in this course, we employ thematic analysis to identify themes in student perceptions that reflect central ideas in these documents. Four aspects of the course arose that likely support these themes and related standards and recommendations. This study generates other questions about the extent to which mathematics content courses focusing on research-based knowledge and practices for PSMTs should naturally elicit solid links to these standards.

Keywords: functions, equations, mathematical knowledge for teaching, inquiry-based learning, standards-based content courses

Introduction

Content courses and methods courses specifically for prospective secondary mathematics teachers (PSMTs) exist in a variety of formats and target a broad range of topics. Professional organizations in the United States such as the Association of Mathematics Teacher Educators (AMTE) and National Council of Teachers of Mathematics (NCTM) provide standards for preparing teachers of mathematics (NCTM Council for the Accreditation of Educator Preparation Standards [NCTM CAEP] 2012; AMTE Standards for the Preparation of Teachers of Mathematics [AMTE SPTM] 2017). In 2012, the Conference Board of the Mathematical Sciences (CBMS) also made recommendations for those preparing future teachers in The Mathematical Education of Teachers (MET) II report which updated the previous 2001 MET I report to respond to increased attention by the mathematics profession on the mathematical education of teachers (CBMS MET II 2012; CBMS MET I 2001). Whereas universities acquiring CAEP accreditation are required to map program elements and assessments to NCTM CAEP (2012) Standards, programs that do not pursue CAEP accreditation may have limited evidence that content courses and methods courses for PSMTs align to national standards. Furthermore, even less may be known about the extent to which these courses address key aspects of the AMTE SPTM or the recommendations in MET II.

MET II (2012) offers suggestions for rethinking mathematics courses to incorporate the mathematical knowledge for teaching that teachers at different levels need. In addition, AMTE SPTM (2017) aims "to engage the mathematics teacher education community in continued research and discussion about what candidates must learn during their initial preparation as teachers of mathematics" (p. 1). In this paper, we identify alignment of a mathematics-content course for PSMTs to MET II recommendations and AMTE SPTM and compare this to themes arising from student interviews regarding their experiences in the course. We also identify key aspects of this course that may contribute to positive associations between researcher-identified connections to the standards and themes arising from student perceptions of the course.

Background

This research is part of a larger project, the *Enhancing Explorations in Functions for Preservice Secondary Mathematics Teachers* Project, which focuses on developing research-based tasks and explorations for use in mathematics courses for PSMTs and creating instructor materials that assist mathematicians and other instructors in using the tasks and explorations in an inquiry-based, active learning environment (Álvarez, Jorgensen, & Rhoads 2019). In the widely-used UTeach (UTeach Institute, n.d.) teacher preparation model in the United States, PSMTs enroll in a required mathematics course where they engage in explorations designed to strengthen and expand their knowledge and understanding of the topics from secondary school mathematics. These materials were designed for use in mathematics courses with similar goals that assume a prerequisite knowledge of second-semester calculus. In particular, a significant goal of these courses focuses on deepening PSMTs understanding of topics related to function (e.g. function versus equation, graphical connections to function patterns, etc.)

Álvarez, Jorgensen, and Rhoads (2019) designed 12 lessons comprising Unit 1 of the course materials with the intent to create powerful learning experiences for PSMTs by inducing cognitive conflict between their existing conceptions of functions and the concepts explored (Loucks-Horsely et al. 2003; Watson & Mason 2007). For example, in a task from Exploration 5.3, PSMTs are challenged to make sense of the familiar equation $y = 2x + 1$ when it arises in a non-familiar context, that is, as the solution set resulting from the intersection of two planes in $\mathbb{R}^3$ (see Figure 1). In this context, they must consider whether $y = 2x + 1$ is also a function. To develop the instructional materials, researchers used a design experiment framework consisting of a cyclic process with "design, enactment, analysis, and redesign" phases (Cobb et al. 2003 p. 5). This was achieved by drawing on existing literature and theory to design and implement instructional materials, and researchers then engaged in the iterative process of collecting data, reflecting on the success of the materials, and re-designing materials.

Although Unit 1 materials were developed with a focus on strengthening and deepening PSMTs understanding of functions, we became interested in how students' perceptions about their experiences in this course, particularly in this unit spanning two-thirds of the entire course, align to standards documents on the mathematical education of teachers. To explore this, two of the authors separately looked for elements in the Fall 2017 implemented lessons on functions (i.e. Unit 1) that link to AMTE SPTM and MET II recommendations. We then identified instances of agreement in the matching of standards and recommendations to the lessons. Based on these shared instances, we determined that the materials supported AMTE SPTM C.1.1, P.3.1, and HS.1 as well as MET II recommendation 1, 2, and 4 (see Table 1). We also matched another statement from the MET II document to the materials (MII56): "…learning mathematical

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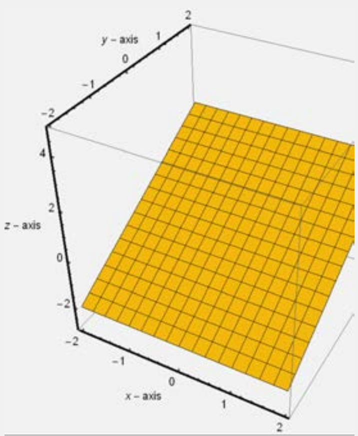
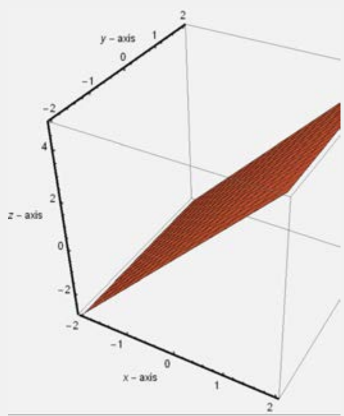
reasoning and actively participating in class will be easier when the learning builds on existing knowledge of high school mathematics" (MET II p. 56).

3. Consider the functions $f(x, y) = y$ and $g(x, y) = 2x + 1$. The graphs of f and g are provided below for reference.

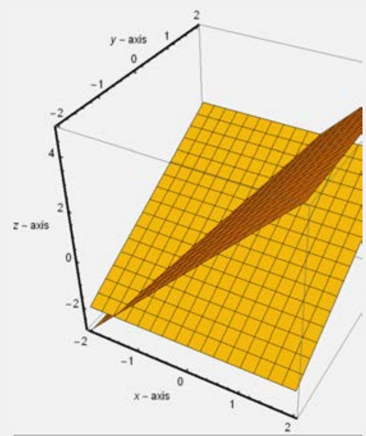
Graph of $z = f(x, y)$



Graph of $z = g(x, y)$



Graphs of $z = f(x, y)$ and $z = g(x, y)$



a. What is the meaning of $f(x, y) = g(x, y)$? Explain.

b. Discuss why $y = 2x + 1$ is an equation (part of your discussion should involve the definition of equation). Is it also a function? Explain.

Figure 1

Exploration 5.3 task focused on meaning of function and equation from course materials

Specifically, we determined that all the lessons in Unit 1 require that prospective teachers have solid and flexible knowledge of core mathematical concepts of function and related procedures that they will teach in high school mathematics, as well as connected horizon content knowledge (Ball et al. 2008; AMTE SPTM pp. 8 & 122; MET II p. 17). We also found that the lessons provide prospective mathematics teachers with experiences that "represent mathematics as a useful, challenging, and interesting discipline" (AMTE SPTM p. 33) and allows them time to engage in sense making about the mathematics they will teach (MET II p. 17). The delivery of the content involves inquiry-based methods and encourages development of mathematical habits of mind (MET II p. 19). Students primarily work in collaborative groups and are able to engage in the mathematics because it builds upon existing knowledge of high school mathematics (MET II p. 56)

The approach this study takes to qualitative inquiry is thematic analysis. While some researchers characterize thematic analysis as a tool used throughout different methods of qualitative analysis, Braun and Clarke (2006), "argue thematic analysis should be considered a method in its own right" (p. 78). Thematic analysis is a method, independent of theory and epistemology, for identifying, analyzing, and reporting themes within data. A theme is identified by the researchers as an idea that captures something important about the data relative to the research questions. Although grounded theory also seeks to identify patterns or themes in the data, the goal of grounded theory is to generate a plausible, relevant theory in response the research questions. Thematic analysis, however, does not bind researchers to the theoretical commitment of producing a theory (Braun & Clarke 2006).

Table 1

AMTE SPTM and MET II Matched Instances in Lessons on Functions in the Course

Standard	Description
AMTE SPTM C.1.1	Well-prepared beginning teachers of mathematics have solid and flexible knowledge of core mathematical concepts and procedures they will teach, along with knowledge both beyond what they will teach and foundational to those core concepts and procedures.
AMTE SPTM P.3.1	An effective mathematics teacher preparation program provides mathematics methods courses or related experiences that represent mathematics as a useful, challenging, and interesting discipline.
AMTE SPTM HS.1	Well-prepared beginning teachers of mathematics at the high school level have solid and flexible knowledge of relevant mathematical concepts and procedures from the high school curriculum, including connections to material that comes before and after high school mathematics and the mathematical processes and practices in which their students will engage. Relevant mathematical concepts include algebra as generalized arithmetic, functions in mathematics, diagrams and definitions in geometry, and statistical models and statistical inference.
MET II Rec. 1	Prospective teachers need mathematics courses that develop a solid understanding of the mathematics they will teach.
MET II Rec. 2	Coursework that allows time to engage in reasoning, explaining, and making sense of the mathematics that prospective teachers will teach
MET II Rec. 4	All courses and professional development experiences for mathematics teachers should develop the habits of mind of a mathematical thinker and problem-solver, such as reasoning and explaining, modeling, seeing structure, and generalizing. Courses should also use the flexible, interactive styles of teaching that will enable teachers to develop these habits of mind in their students.
METII56	Finally, learning mathematical reasoning and actively participating in class will be easier when the learning builds on existing knowledge of high school mathematics.

Methods

This study took place at a large, urban university in the southwestern United States with an on-campus enrollment greater than 42,000 students. Students of color from groups historically disenfranchised in STEM in the United States comprise more than 35% of the student body. During the Fall 2017 implementation of the project materials, 22 of 24 PSMTs enrolled in the course chose to participate in the research process. The course met twice per week for 80-minute class periods over a 15-week semester. The course focused on inquiry-based learning and activities with students mostly working in small groups assigned by the instructor. As stated above, the first part of the course, Unit 1, lasted two-thirds of the semester or ten weeks with an exam on Unit 1 in week 11. Approximately 36% of the participants were Hispanic or African American and approximately 27% Asian American, with non-Hispanic white students and others comprising approximately 64% of the participants. Of these, there were 15 female students and 7 male students.

A copy of all written work was collected for each participant including written explorations, daily journals, homework, labs, exams, and a midterm project. In addition, the instructor kept planning logs and participated in post-lesson interviews to reflect on lesson implementation. Researchers also developed and administered a pre- and post-assessment on function knowledge to all participants, and 11 PSMTs completed individual, task-based interviews to explore their understanding of functions further. Five of the interview participants were Hispanic or African American and three were Asian American.

The individual, task-based participant interviews were conducted and video recorded during a one-hour session following the completion of the course unit on functions during weeks ten through thirteen of the semester. Within the interview, students completed tasks that were related to mathematical ideas they explored in the course, and students also answered questions related to their perceptions of the class learning environment. In this paper, we will focus on portion of the interviews associated with students' perceptions of the learning environment. Each participant answered four questions pertaining to perceptions of the course:

1. What do you think about the inquiry format of this course?
2. Is this format something you would integrate into your own teaching?
3. What features of the class format or environment do you think enhanced your learning?
4. What features of the class format or environment do you think detracted from your learning?

If needed, the researcher would ask follow-up questions to encourage the participant to clarify or extend a response.

Upon completion of the individual, task-based participant interview from Fall 2017, the video recordings were blinded and then transcribed by one of the authors using NVivo 11. Using thematic analysis, she identified patterns within the student interview data that reflected central ideas in the AMTE SPTM and MET II recommendations. Themes were identified using a theoretical or 'top down' approach. 'Theoretical' thematic analysis is driven by the researcher's interest in the area, and the coding process is carried out with a specific research question in mind. As we specifically sought to explore how students' perceptions about their experiences align to these standards documents, themes were identified with a theoretical approach (Braun & Clarke, 2006).

From the analysis of the student interview data, four themes in students' perceptions of the course were identified: *Value in Different Approaches, Perseverance and Problem Solving, Importance of Precision, and Extended Understanding of Material. Interactive Learning Environment* was initially identified as a theme; however, it was excluded from further analysis because we concluded that it may have been prompted by the first interview question in which participants were specifically asked about the inquiry nature of the course.

A participant's statement was coded as *Value in Different Approaches* if it indicated understanding of the benefit of others' perspectives in approaching the mathematics. This is seen in Ben's answer to the features of the class learning environment that enhanced his learning. "…it is more fun when you talk to your classmates, my classmates. Sometimes they have like wonderful ideas that I had never thought of, and sometimes it is just so simple, but I think [it's] so hard."

Remarks that illustrated participants' perspectives on the perseverance needed to reach resolutions to the explorations gave rise to the code *Perseverance and Problem Solving*. During Jordan's interview, she expressed, "…like every day it was, 'Ok, go home and think about [the lesson].' You know the journal is supposed to help you think it through, but then at the same time, I don't know if I'm thinking through it right. I don't, I don't even think that I finished it because I didn't even know how to, how to think about it." Norma stated, "Sometimes it is frustrating, but I feel like it is more rewarding when you yourself, discover, or like yeah, when you yourself, discover, or find out the causes behind it. Rather than being told the answer." While Jordan describes her unwillingness to continue engaging in the problem solving process, Norma says she prefers to persevere and discover the answer herself.

Importance of Precision describes instances where participants learned a difference between two ideas or terms that they did not previously know. For example, Amber is discussing functions and equations and says, "From an outsider's perspective it seems like a simple difference, if there's even a difference at all, but then you actually start analyzing it…and you realize that the differences are actually huge and the way you word things does matter especially when you're teaching it."

When a participant expressed gaining a deeper understanding of a topic through interaction with the materials in the course, this was coded as *Extended Understanding of Material*. This is seen when Josie is discussing her thoughts on the format of the course and states, "I think that one of the coolest things that we did was when we did the complex number line of the parabola. Like the solutions. That just blew my mind. I was really happy after I learned that, and I was like, 'Why don't we learn this in school?'" Norma also says that although all the concepts in this course had been taught to her in middle school or high school, in this class students "…are thinking in more depth to it, and answer[ing] the question as to how and why. And [she] feel[s] like if [students] understand how and why, in the future, [they'll] be better at explaining it in simpler terms."

Findings

While the primary focus in developing the course materials in Unit 1 rested in strengthening and deepening PSMTs understanding of functions, we also found that the explorations and tasks in the lessons connected to the AMTE SPTM and MET II recommendations in multiple ways. Not only are the recommendations and standards we connected to lesson tasks represented within the interview themes, but other ties to the standards and recommendations that we had not identified in the course materials also emerged in the participants' interview data (see Table 2).

The *Perseverance and Problem Solving* theme ties to MET II recommendations 2 and 4 that we identified in the analysis of the materials. As this theme relates to students' perspectives on the nature of the course, student responses indicate that the coursework "allows time to engage in reasoning, explaining, and making sense of mathematics" (MET II, 2012, p.17) and encourages development of "the habits of mind of a mathematical thinker and problem-solver" (p.19).

MET II recommendation 1 and AMTE SPTM C.1.1 and HS.1 describe the importance of PSMTs having a solid understanding of the mathematics they will teach as well as horizon content knowledge. As seen in Table 1, we identified alignment of these recommendations and standards with the course materials. These also relate to the student interview data associated with the themes *Importance of Precision* and *Extended Understanding of Materials*. Both these themes and the identified standards and recommendations describe PSMTs' gaining new insight into concepts, with particular attention to conceptual precision and mathematical concepts believed to have been already fully understood. This plausibly enhances understanding of the mathematics they will teach as well as broad connections in the development of the mathematical ideas.

Table 2

Examining Ties Between Themes, Standards and Recommendations

Themes from Student Interview Data	**Value in Different Approaches**	**Perseverance and Problem Solving**	**Importance of Precision**	**Extended Understanding of Material**
Description of Theme	Student indicates understanding of the benefits of others' perspectives in problem solving.	Student provides reflections or opinions about perseverance in problem solving.	Student describes an instance he or she learned a difference between ideas or terms previously thought to be the same.	Student conveys an extended understanding of material was gained through interaction with course materials.
Related AMTE SPTM standards and MET II recommendations from researcher analysis of course materials		Rec. 2 Rec. 4	C.1.1 HS.1 Rec. 1	C.1.1 HS.1 Rec.1
AMTE SPTM and MET II standards that emerged directly from themes in student interviews (not previously identified from course materials)	C.1.5	C.1.2 C.1.3	C.1.2	

There were two recommendations from the documents that we linked to the course that were not overt in the student interview data – AMTE SPTM P.3.1 and MII56. AMTE SPTM P.3.1 states that a "mathematics teacher preparation program provides mathematics methods courses or related experience that represent mathematics as a useful, challenging or interesting discipline" (AMTE SPTM 2017 p. 33). Also, MII56 says, "learning mathematical reasoning and actively participating in class will be easier when learning builds on existing knowledge of high school mathematics" (MET II p. 56). These recommendations did not explicitly relate to the themes that emerged in the student interview data.

Although we did not necessarily expect other recommendations from the AMTE SPTM and MET II documents to align to course materials, evidence of AMTE SPTM C.1.2., C.1.3, and C.1.5 emerged in students' perceptions of the course. AMTE SPTM C.1.5 indicates that PSMTs "value varied approaches in solving a problem, recognizing that engaging in mathematics is more than finding an answer" (AMTE SPTM 2017 p. 10). As *Value in Different Approaches* is characterized by students' indications that they understand the value in others' ideas in the problem solving process, this theme directly relates to AMTE SPTM C.1.5.

In addition to corresponding to recommendations that researchers identified in their analysis of the course materials, *Perseverance and Problems Solving* and *Importance of Precision* also relate to AMTE SPTM C.1.2. The explanation of AMTE SPTM C.1.2 expresses PSMTs are to "use mathematical language with care and precision" which coincides with the description of *Importance of Precision* (AMTE SPTM p. 9). AMTE SPTM C.1.2 also states that PSMTs should "regard doing mathematics as a sense-making activity that promotes perseverance, problem posing, and problem solving" (AMTE SPTM p. 9). While student responses coded as *Perseverance and Problem Solving* were not always positive opinions, they do indicate that the course gave student opportunities to engage with mathematics as a sense-making activity and encouraged them to persevere during problem solving.

Another standard prevalent in the student interview data is AMTE SPTM C.1.3. PSMTs are described in AMTE SPTM C.1.3 as "able to solve sophisticated mathematical problems with effort" and someone who understands that "one's success in mathematics depends on a productive disposition toward the subject and on hard work" (AMTE SPTM p. 9). This standard also relates to the *Perseverance and Problem Solving* theme. Although some students were not always appreciative of the way in which the course pushed their mathematical understanding and required more work than they were perhaps accustomed to, the emergence of *Perseverance and Problem Solving* as a theme indicates that students recognized that the course promoted these ideas.

Discussion

As Álvarez, Jorgensen, & Rhoads (2019) did not specifically design course materials or participant interviews with the intention of linking to specific ideas from the AMTE SPTM and MET II documents, our findings gave rise to the question: What aspects of the course contributed to students' developing the perspectives identified in these themes? To gain insight into this question, we examined each theme, and within each theme looked for commonalities in the comments made by students.

The *Importance of Precision* theme consists of statements from three participants in which they identified that through the course they learned nuances between mathematical ideas previously unknown. In each of these statements, students specifically spoke of learning the difference between the terms function and equation. Deepening students' understanding of the relationship between the terms function and equation was a goal of the researchers in developing course materials. It appears, then, that the emergence of *Importance of Precision* and the connected standards and recommendations (MET II recommendation 1 and AMTE SPTM C.1.1, C.1.2, and HS.1) may be related to researchers providing students opportunities to engage in activities that could create cognitive conflicts pertaining to common misconceptions PSMTs hold about functions and equations.

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Ten participants made statements that were coded as *Perseverance and Problem Solving*. In these statements, seven students made references to the open-ended nature of the course such as not being provided with definitive closure at the end of every class period. Six students also specifically discussed the journals that were assigned to help students reflect on the lessons outside of class. Considering these similarities, the statements coded as *Perseverance and Problem Solving* and related standards and recommendations (MET II recommendation 2 and 4, and AMTE SPTM C.1.2 and C.1.13) may be associated with the inquiry nature of the course along with the intentional choice by researchers to encourage individual reflection by the students.

The collaborative environment in the classroom provided students with many opportunities to engage in the open-ended tasks and explorations in small groups. The *Value in Different Approaches* theme contains statements made by nine participants in which they expressed seeing the benefit of others' perspectives when approaching mathematics. Of these statements classified as *Value in Different Approaches*, six participants made explicit references to the ability to work in groups or discuss topics with their classmates. Thus, we assert that the collaborative environment in the course supported this theme arising from the participants' interview data and associated standards and recommendations (AMTE SPTM C.1.2., C.1.3, and C.1.5).

Eight participants also provided statements later coded as *Extended Understanding of Material*. Moreover, four of these participants made specific comments about the depth of the material that was covered in the course. Gabriel, for example, stated, "... before going into that class [it's] just kind of function is just this and that, and then like we go into the class and [we] learn all these definitions and [we go] really deep into it." These statements about the depth of the material covered in this course indicate that the *Extended Understanding of Material* theme and ties identified to related standards and recommendations (MET II Rec. 1 and AMTE SPTM C.1.1, and HS.1) may derive from the design of the materials and structure of the classroom environment. These course components purposefully provided PSMTs opportunities to engage with secondary school mathematics topics on a level deeper than previously encountered and to integrate content knowledge from their early college courses in mathematics.

Conclusion

We believe the following key aspects of the course support the emergence of the four themes identified in PSMTs' perceptions of the course. The course structure and environment provided opportunities to

- engage in purposefully designed activities that elicit cognitive conflicts related to mathematical conceptions PSMTs may hold (da Ponte & Chapman 2008; Henningsen & Stein 1997; Vinner 1991),
- explore the depth of secondary school mathematics concepts and their connections to university-level mathematics (Ball et al. 2008; Laursen, Hassi, & Hough 2016),
- experience inquiry-based explorations with integrated, intentional opportunities for PSMTs' reflection on their understandings (da Ponte & Chapman 2008; Laursen et al. 2016), and
- collaborate on open-ended mathematical tasks in a classroom where working in small groups is the norm (da Ponte & Chapman 2008; Laursen et al. 2016; Vygotsky, 1978).

These components underscore the important role of deep and connected mathematical content coupled with instructional choices that allow students to engage meaningfully and collaboratively with the mathematics. Findings from other studies identify similar course components in effective learning environments. For example, in their review of studies on preservice mathematics teachers (PMTs), da Ponte and Chapman (2008) contend that courses for PMTs should be exploratory in nature, encourage reflection on mathematical ideas, and focus on school mathematics while emphasizing connections to mathematical ideas. Laursen et al. (2016) also found in their study that PMTs' development of mathematical knowledge for teaching is supported by inquiry-based learning (IBL). Furthermore, they identify the essential features of an IBL course as collaboration with peers and the ability to engage with mathematics deeply (Laursen et al. 2016).

The alignment between what an instructor identifies as learning goals for a course and whether students perceive the same learning goals may sometimes be problematic. The materials on functions in Unit 1 of this course aimed to deepen PSMTs' understanding of functions by attending to topics and experiences that address faulty conceptions of the mathematics as well as critical conceptions of the mathematics for teaching (e.g. Carlson, Oehrtman, & Engelke 2010; Chazan & Yerushalmy 2003; Cooney, Beckman, Lloyd, Wilson, & Zbiek 2010; Even 1993; Hitt 1998; Kieran 1981; Knuth, Stephens, McNeil, & Alibali 2006; Leinhardt, Zaslavsky, Stein 1990; Presmeg 2006; Wilson 1994). However, retroactively analyzing the materials for their fidelity to the AMTE SPTM and MET II recommendations revealed that the intent to create research-based materials might also have helped support the integration of proficiencies for future secondary mathematics teachers that have been identified in these standards and recommendations. In exploring student perceptions of the course and materials, the themes identified in their responses linked to most of the standards and recommendations we identified in the materials. This correspondence between researcher-perceived connections and themes derived from student perceptions to the AMTE SPTM and MET II recommendations provides evidence that these proficiencies surfaced in the course to some extent.

Whereas the AMTE SPTM (2017) state that "Although these proficiencies are grounded in available research, in many areas that research is not yet sufficient to determine the specific knowledge, skills, and dispositions that will enable beginning teachers to be highly effective in their first years of teaching," our intent to create research-based materials for PSMT's does align with several areas in AMTE SPTM and MET II recommendations (p.1). As mathematics teacher educators teach and create content-based mathematics courses for PSMTs, attending to critical course components related to deep connections and pivotal understandings of mathematics for teaching as well as course structures that promote inquiry-based learning and collaboration may be commonplace. However, examining the courses for ways in which the AMTE SPTM link to the course content and gathering evidence related to PSMTs' identifying these proficiencies in their experiences in the course may lead to a better understanding of key aspects of these courses that support the AMTE SPTM.

We identified four generalizable course components that emerged when student perspectives aligned with national standards. As mathematics teacher educators examine their own content-based courses for PSMTs for alignment to teacher preparation standards, would weak alignment indicate possible problems with the mathematics content not reflecting research-based content associated with important mathematics for teaching? What would it mean if student perceptions do not intersect with instructor-perceived connections to the standards? The outcomes from this retrospective examination of these course materials with respect to national teacher preparation standards and recommendations may indicate that courses for PSMTs focusing on research-based knowledge and practices should naturally elicit solid links to the standards. However, exploring PSMTs corresponding experiences and understandings to corroborate these links remains essential.

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