Pre-Service Elementary School and Secondary Mathematics Teachers' Van Hiele Levels and Gender Differences

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Abstract

The aim of this study was to find and compare the pre-service elementary school and secondary mathematic teachers' reasoning stages in geometry. There were a total of 281 pre-service teachers, 125 elementary school teachers and 156 secondary mathematics teachers, involved in the study. The researcher employed a multiple-choice geometry test. This test was developed to find out one's geometric reasoning levels. After the collection of the data, the independent samples t-test with $\alpha = .05$ was used to analyze the data. The study found that there was no statistically significant difference in regard to the reasoning stages between the pre-service elementary school and secondary mathematics teachers, and that although there was a difference with reference to van Hiele levels between male and female pre-service secondary mathematics teachers favoring males, there was no sex-related difference found between male and female pre-service elementary school teachers.

Key words: *Van Hiele levels; pre-service mathematics teachers; gender; geometry*

Introduction

There has been a growing interest in the area of teaching and learning geometry since the mid 1980s (e.g., Crowley, 1987; Gutierrez, Jaime, & Fortuny, 1991; Clements & Battista, 1990; Mason, 1997; Lappan, Fey, Fitzgerald, Friel, & Phillips, 1996). The National Council of Teacher of Mathematics (NCTM) (2000) recommends that new ideas, strategies, and research findings be utilized in teaching in order to help students overcome their difficulties in learning mathematics. Besides, knowledge of theoretical principles gives teachers an opportunity to devise practices that have a greater possibility of succeeding (e.g., Swafford, Jones, & Thornton, 1997).

Research has shown that many students are having difficulties in learning geometry and showing poor performance in mathematics classrooms (e.g., Gutierrez, Jaime, & Fortuny, 1991; Yetkin, 2003; Halat, 2006/2007). It seems that teacher's mathematical and pedagogical content knowledge plays vital roles in student learning. In addition, Chappell (2003) claims that the mathematics teachers who have insufficient mathematics knowledge are hired to teach at middle or high schools. According to Stipek (1998), teacher is one of the most important factors in student learning because students spent most of their time in the schools. Therefore, teacher education programs should be carefully designed and updated based on research findings.

The Van Hiele Theory

Based on over twenty years of research, the van Hiele theory is a well-known theory in learning and teaching geometry, structured and developed by Pierre van Hiele and Dina van Hiele-Geldof between 1957 and 1986. It has its own reasoning stages and instructional phases in geometry.

The van Hieles described five levels of reasoning in geometry. These levels, hierarchical and continuous, are level-I (Visualization), level-II (Analysis), level-III (Ordering), level-IV (Deduction), and level-V (Rigor) (van Hiele, 1986). The descriptions of the five levels can be seen in different places (e.g., Fuys, Geddes & Tischler, 1988; Knight, 2006). The existence of level-0 is the subject of some controversy (e.g., Usiskin, 1982; Burger & Shaughnessy, 1986). Van Hiele (1986) does not talk about or acknowledge the existence of such a level. However, Clements and Battista (1990) talked about the existence of a level–0 called pre-recognition. Clements and Battista (1990) have described and defined ***level-0 (Pre-recognition)*** as "Children initially perceive geometric shapes, but attend to only a subset of a shape's visual characteristic. They are unable to identify many common shapes" (p.354). For example, learners may see the difference between triangles and quadrilaterals by focusing on the number of sides the polygons have but not be able to distinguish between any of the quadrilaterals (Mason, 1997).

Research about the van Hiele Theory

Research has been completed on various components of this teaching and learning model. For instance, Wirszup (1976) reported the first study of the van Hiele theory, which attracted educators' attention at that time in the United States. In 1981, Hoffer worked on the description of the levels. Usiskin (1982) affirmed the validity of the existence of the first four levels in geometry at the high school level. In 1986, Burger and Shaughnessy focused on the characteristics of the van Hiele levels of development in geometry. They stated "students in the study who appeared to reason at different levels used different language and different problem solving processes on the tasks"(p.46). Furthermore, they said that students showed different levels of reasoning on different tasks. Fuys, Geddes, and Tischler (1988) examined the effects of instruction on a student's predominant van Hiele level. Senk (1989), Mason (1997), and Gutierrez & Jaime (1998) evaluated and assessed the geometric abilities of students as a function of van Hiele levels. The study of Gutierrez, Jaime, & Fortuny (1991) with 9 eighth-grade pupils and 41 future primary school teachers was on an alternative way of analyzing the van Hiele levels of geometric thinking in the solid geometry. According to their study, most future primary teachers' van Hiele levels were level-I (recognition) and –II (analysis), but none of the participants showed level-IV (deduction) reasoning stage.

Mayberry (1983) conducted a study with 19 pre-service elementary school teachers. The tasks employed in her study were designed for the first four levels including seven geometric concepts that were squares, right triangles, isosceles triangles, circles, parallel lines, similarity, and congruence. According to the results of her study (1983), "the finding that 70% of the response patterns of the students who had taken high school geometry were below level-IV (deduction)" (p.68-69). In addition, the response of patterns showed that students who took part in the study were not at the suitable level to understand formal geometry, and that the instruction they had taken had not brought them to level IV (Deduction). The students' responses implied that the typical student in the study was not ready for a formal deductive geometry course (Mayberry, 1983).

Moreover, there have been some studies with pre-service elementary and secondary mathematics teachers regarding their reasoning stages in geometry. For instance, Knight (2006) conducted a research exercise with a total of 68 pre-service mathematics teachers, 46 elementary and 22 secondary. She found that the pre-service elementary and secondary mathematics teachers' reasoning stages were below level-III (informal deduction) and level-IV (deduction), respectively (Knight, 2006). Her findings are surprising because the van Hiele levels of pre-service elementary and secondary mathematics teachers are lower than the level expected of students completing grade 8 and grade 12, respectively. These results are consistent with the findings of Gutierrez, Jaime, & Fortuny (1991), Mayberry (1983), Duatepe (2000), and Durmuş, Toluk, & Olkun (2002). In all of these studies, none of the pre-service elementary and secondary mathematics teachers showed a level-V (Rigor) reasoning stage in geometry. Clearly, this is not a desirable outcome in teacher education.

It is also shown that reform–based or NSF-funded standards-based curricula (e.g., Connected Mathematics Project, MATH Thematics, University of Chicago School Mathematics Project, Core-Plus Mathematics Project, and the New Zealand Numeracy Project (NZNP)) have positive effects on students' learning of mathematics more than conventional ones (cf., Fuson, Carroll, & Drueck, 2000; Romberg & Shafer, 2003; Reys, Reys, Lapan, Holliday, & Wasman, 2003; Senk & Thompson, 2003; Young-Loveridge, 2005). Moreover, according to Halat (2006), reform–based geometry curricula had a very favorable impact on the acquisition of the van Hiele levels and motivation in learning geometry.

Burger & Shaughnessy (1986) and Halat (2007) found mostly level-I reasoning in grades K-8 while Fuys et al. (1988) found no one performing above level-II in interviewing sixth and ninth grade average and "above average" students, which supports the idea that most younger students and many adults in the United States reason at levels-I (Visualization), –II (Analysis), -III (Ordering) and –IV (Deduction) of van Hiele theory (e.g., Usiskin, 1982; Hoffer, 1986; Mayberry, 1983; Knight, 2006). Mayberry (1983) and Fuys, Geddes, & Tischler (1988) stated that content knowledge in geometry among pre-service and in-service middle school teachers is not adequate. There are many factors, such as gender, peer support, age, type of mathematics course, instruction, and so forth that appear to be affecting pre-service mathematics teachers' or college students' performance and motivation in mathematics.

Gender Differences in Mathematics

Forgasız (2005) for whom gender is still a matter of concern in mathematics education argued that it is significantly important to include gender as a variable in research analysis even if it is not the main focus of a study. Moreover, according to Armstrong (1981), Ethington (1992), Grossman & Grossman (1994) and Lloyd, Walsh & Yailagh (2005), gender is an important factor in learning mathematics. These arguments motivated the researcher to examine this variable.

Over the past few decades, research has shown that although there is a difference between the achievement of male and female students in many content areas of mathematics, such as spatial visualization, problem solving, computation, measurement applications and so forth (e.g., Jones, 1989; Grossman & Grossman, 1994; Lloyd, Walsh & Yailagh, 2005). For instance, according to Armstrong (1981), female students performed better at computation and spatial visualization than males. However, according to Fox and Cohn (1980), there was a significant sex difference in mathematics achievement at the high school level. Males' performance was better than that of females on the Scholastic Aptitude Test in mathematics. Similarly, Smith & Walker (1988) found that there were statistically significant sex-related differences in favor of male students in geometry at the tenth grade level.

However, in recent years a considerable decrease can be seen in the gender gap between male and female students' attitudes towards mathematics (e.g., Friedman, 1994; Fennema & Hart, 1994). For example, Fennema & Hart (1994) claimed that interventions could achieve equity in learning mathematics. Likewise, according to Halat (2006), instruction influenced by the van Hiele theory-based curricula may cause changes in girls' negative attitudes towards mathematics courses because reform-based works in mathematics teaching and learning, such as the New Zealand Numeracy Projects (NZNP) (Young-Loveridge, 2005) and standard-based curricula, such as "Everyday Math" and "MathThematics" have positive impacts on student achievement and motivation in mathematics (e.g., Billstein & Williamson, 2003; Chappell, 2003).

According to Hyde, Fennema & Lamon (1990) and Malpass, O'Neil & Hocevar (1999), there is also a considerable increase in the gender gap among gifted or high scoring students on mathematics tests. There are many factors, such as prior achievement, value, stereotyping mathematics as a male domain, parental support, teacher-care, peer-support, instruction, and curriculum appearing to play vital roles in the sex differences between boys and girls in mathematics (e.g., Becker, 1981; Ethington, 1992; Grossman & Grossman, 1994; Fan & Chen, 1997).

In short, it is clear that gender is an important factor affecting student performance in mathematics and research findings are also varied in this issue.

The Purpose of the Study

This current study was to find out the van Hiele levels of the pre-service elementary school teachers and secondary mathematics teachers in geometry and examine the effects of gender on the reasoning stages. In particular, the following questions guided this study:

1. What difference, if any, exists in terms of geometric reasoning stages between the pre-service elementary school and secondary mathematics teachers?

2. Is there a difference in regard to the van Hiele levels between male and female pre-service elementary school and secondary mathematics teachers, respectively?

Method

Participants

In this study the researcher followed the "convenience" sampling procedure defined by McMillan (2000), where a group of participants is selected because of availability. The number of the participants was a total of 281 pre-service teachers. There were 125 (44.5%) pre-service elementary school teachers, 68 (54.4%) female and 57 (45.6) male, and 156 (55.5%) pre-service secondary mathematics teachers, 72 (46.2%) female and 84 (53.8%) male, involved in the study.

The study took place in a university located in central Turkey. The pre-service secondary mathematics teachers had taken Euclidean geometry before coming to the university. Therefore, the Department of Mathematics in the university does not offer a course regarding Euclidean geometry, but the students should take higher level mathematics and geometry courses. In other words, the department assumes that students have strong geometry knowledge in Euclidean geometry. However, the pre-service elementary school teachers are required to take a geometry course for a semester. Thus, all pre-service elementary school teachers involved in the study took the geometry course before participating in the current study.

Data Sources

The researcher gave participants a geometry test called *van Hiele Geometry Test (VHGT)*. The VHGT was administered to the participants by the researcher during a single class period. The *van Hiele Geometry Test (VHGT)* consists of 25 multiple-choice geometry questions. The VHGT was taken from the study of Usiskin (1982) with his written permission. The VHGT is designed to measure one's van Hiele levels in geometry. This test was translated to Turkish language by the author. Five mathematicians reviewed Turkish version of VHGT in terms of its language and content.

Test Scoring Guide

In this study, the 1-5 scheme was used for the levels. This scheme allows the researcher to use level-0 for students who do not function at what the van Hieles named the ground or basic level. It is also consistent with Pierre van Hiele's numbering of the levels. For this report, all references and all results from research studies using the 0-4 scale have been changed to the 1-5 scheme.

All participants' answer sheets from VHGT were read and scored by the investigator. All participants got a score referring to a van Hiele level from the VHGT guided by Usiskin's grading system.

"For van Hiele Geometry Test, a student was given or assigned a weighted sum score in the following manner:

- 1 point for meeting criterion on items 1-5 (level-I)
- 2 points for meeting criterion on items 6-10 (level-II)
- 4 points for meeting criterion on items 11-15 (level-III)
- 8 points for meeting criterion on items 16-20 (level-IV)
- 16 points for meeting criterion on items 21-25 (level-V)" (1982, p. 22)

Analysis of Data

The data were responses from students' answer sheets. In the process of the assessment of participants' van Hiele levels, the criterion for success at any given level was four out of five correct responses. The independent samples t-test with $\alpha = .05$ was employed to analyze the data and to examine the reasoning stages of pre-service elementary school and secondary mathematics teachers and the pre-service male and female teachers in each group. The researcher also constructed a frequency table to get in-depth information about the van Hiele levels distribution for the pre-service elementary school and secondary mathematics teachers.

Results

1. What difference, if any, exists in terms of geometric reasoning stages between the pre-service elementary school and secondary mathematics teachers?

Table 1 indicates the descriptive statistics and the independent samples t-test for the pre-service elementary school and secondary mathematics teachers. It demonstrates that the mean score of the pre-service secondary mathematics teachers (2.29) is numerically higher than that of the pre-service elementary school teachers (2.22). However, according to the independent samples t-test, the mean score difference in terms of reasoning stages between these two groups was not statistically significant [t= .71, p= .47 > .05]. This means that there is no statistically significant difference as in geometric reasoning levels between the pre-service elementary school and secondary mathematics teachers. This finding is surprising because the pre-service secondary mathematics teachers are expected to attain higher geometric reasoning levels than the pre-service elementary school teachers.

Table 1

Descriptive Statistics and the Independent Samples T-Test for Pre-service Elementary & Secondary Math Teachers' van Hiele Levels

Groups	N	van Hiele Geometry Test					
		M	SD	SE	df	t	p
A	125	2.22	.91	.08	264.27	.71	.47
B	156	2.29	.93	.07			
Total	281						

Note. A: Pre- Service Elementary School Teachers, B: Pre-Service Secondary School Math Teachers

According to Burger & Shaughnessy (1986), the progress through the levels is continuous and not discrete. In spite of the fact that students generally are assigned to a single van Hiele level, there may be students who cannot be assigned to a single van Hiele level. Gutierrez, Jaime, & Fortuny (1991) used a 100 - point numerical scale to determine the van Hiele levels of students who reason between two levels. This numerical scale is divided into five qualitative scales: "'Values in interval' (0%, 15%) means 'No Acquisition' of the level. 'Values in the interval' (15%, 40%) means 'Low Acquisition' of the level. 'Values in the interval' (40%, 60%) means 'Intermediate Acquisition' of the level. 'Values in the interval' (60%, 85%) means 'High Acquisition' of the level. Finally, 'values in the interval' (85%, 100%) means 'Complete Acquisition' of the level'" (p. 43).

The mean scores 2.22 and 2.29 of the pre-service elementary school and secondary mathematics teachers can be explained with the scale described above. The scores .22 and .29 can be placed into the interval named "Low Acquisition" of the upper level. In other words, the participants in both groups completed the level-II (Analysis), but they have not attained the level-III (Ordering). At this level (Analysis), students analyze figures in terms of their components and relationships among components and perceive properties or rules of a class of shapes empirically, but properties or rules are perceived as isolated and unrelated. A student should recognize and name the properties of geometric figures (e.g., Fuys et al. 1988).

Table 2 shows the pre-service elementary school and secondary mathematics teachers' van Hiele levels distribution. Even though none of the pre-service elementary school teachers showed level-V (Rigor) reasoning stages, they indicated the rest of van Hiele levels in different percentiles. Mostly they attained at levels- II (Analysis) (36%) and -III (Ordering) (44%) geometry knowledge. This is in line with the findings of Durmuş, Toluk, & Olkun (2002). Besides, there were some showing level-0 (Pre-recognition) (7.2%) geometry knowledge on the test. On the contrary, there were some pre-service secondary mathematics teachers who attained level-V (Rigor) (1.9%). Similarly, most of the pre-service elementary teachers showed level-II (Analysis) (36.5%) and –III (Ordering) (35.9%) geometry knowledge on the test. These findings are interesting because the pre-service secondary mathematics teachers are expected to have higher reasoning stages than the pre-service elementary school teachers. Table 2 shows that the pre-service elementary school teachers are in a better shape than the others in term of geometry knowledge.

Table 2

Frequency Table for Pre-service Elementary & Secondary Math Teachers' van Hiele Levels

Groups	N	Level-0	Level-I	Level-II	Level-III	Level-IV	Level-V
		%	%	%	%	%	%
A	125	7.2	11.2	36	44	1.6	0
B	156	0	21.2	36.5	35.9	4.5	1.9
Total	281						

Note. A: Pre- Service Elementary School Teachers, B: Pre-Service Secondary School Math Teachers

2. Is there a difference in regard to the van Hiele levels between male and female pre-service elementary and secondary mathematics teachers, respectively?

Table 3 presents the descriptive statistics and independent sample t-test for male and female pre-service elementary and secondary mathematics teachers. According to table 3, the mean score of male pre-service secondary mathematics teachers' reasoning stages (2.49) is higher than that of females (2.07). This difference is statistically significant between male and female pre-service secondary mathematics teachers. This was in favor of males [$t= 2.90$, $p= .004 < .05$]. The 2.49 mean score of males can be placed to the third interval scale called ‘Intermediate Acquisition’ of the level, level-III (Ordering). Similarly, the 2.07 mean score of females can be placed into the first interval scale called ‘No acquisition of the level, level-III (Ordering).

On the other hand, although the mean score of male pre-service elementary school teachers’ thinking levels (2.30) is numerically higher than that of female pre-service elementary school teachers (2.15), this difference is not statistically significant [$t= .905$, $p= .36 > .05$]. In other words, there was no sex-related difference found in terms of reasoning stages between male and female pre-service elementary school teachers. Both 2.30 and 2.15 mean scores of males and females can be placed into the second interval scale named ‘low acquisition of the level, level-III (Ordering).

Table 3

Descriptive Statistics and the Independent Samples T-Test for Pre-service Secondary Mathematics Teachers’ van Hiele levels by Gender

Groups	N	van Hiele Geometry Test					
		M	SD	SE	df	t	p
Male[1]	84	2.49	.91	.09	151.92	2.90	.004
Female[1]	72	2.07	.87	.10			
Male[2]	57	2.30	.86	.11	123	.905	.36
Female[2]	68	2.15	.98	.11			

Note. [1] : Pre-service Secondary Mathematics Teachers
[2] : Pre-service Elementary School Teachers

Discussion and Conclusion

The study showed that the participants attained all five van Hiele levels in different percentiles. In particular, there were some showing level-V (Rigor) reasoning stages. This supports the claim of Usiskin (1982), Fuys, Geddes & Tischler (1988), and Gutierrez & Jaime (1998) stating that the last van Hiele level, Rigor, is more appropriate for college students.

The study pointed out that almost eighty-two percents of the pre-service elementary school teachers’ geometric reasoning stages were at or above level-II (analysis). This means that the pre-service elementary school teachers have adequate content knowledge in geometry to teach at elementary schools. NCTM (2000) stated that elementary school students should be able to analyze properties and characteristics of geometric shapes, to make and test conjectures about geometric properties and relationships. Moreover, it supports the finding of Mayberry (1983) stating that pre-service elementary school teachers showed mostly level-III (ordering or informal deduction) geometry knowledge. But, this result is not lined up with the finding of Gutierrez, Jaime, & Fortuny (1991) pointing out that mostly the future primary school teachers showed levels-I (visualization) and –II (analysis) thinking.

On the other hand, the study documented that forty-two percents of the pre-service secondary mathematics teachers’ reasoning stages were at or above level-III (ordering). These pre-service mathematics teachers were expected to teach geometry to the high school students who are supposed to attain at least level-III (ordering) or above. In other words, high school students should know the properties of figures, and interrelate previously discovered properties and rules by giving informal arguments. They should understand and recognize logical implications and class inclusions. In particular, they should prove theorems deductively, construct proofs, and they should understand the role of axioms and definitions

(NCTM, 2000). It seems that many of the pre-service secondary mathematics teachers have insufficient geometry knowledge to teach at high school level. This is consistent with the finding of Knight (2006) who claimed that pre-service secondary mathematics teachers' reasoning stages were below level-IV (deduction) that is an expected level of 12^{th} graders. Obviously, this is not a desirable outcome in teacher education. Because, Chappell (2003) and Fuys, Geddes, and Tischler (1988) expressed that one of the main reasons behind students' poor performance in mathematics at middle or high school level was hiring mathematics teachers who have inadequate content knowledge in mathematics. One would expect that pre-service secondary school mathematics teachers' van Hiele levels should be higher than that of pre-service elementary school teachers. However, according to the result of this study, there was no statistically significant difference as in the geometric reasoning stages between the two groups.

The study also indicated that although there was no statistically significant difference in terms of reasoning stages between male and female pre-service elementary teachers, a statistically significant sex-related difference was found among the pre-service secondary mathematics teachers favoring males. The research findings about gender differences in mathematics have been varied (c.f., Fennema & Carpenter, 1981; Armstrong, 1981; Smith & Walker, 1988). Over the past few decades, research has documented that although there is a difference between the achievement of male and female students in many content areas of mathematics, such as spatial visualization, problem solving, computation, measurement applications and so forth (e.g., Jones, 1989; Grossman & Grossman, 1994; Lloyd, Walsh & Yailagh, 2005), in recent years a considerable decrease can be seen in the gender gap between male and female students' attitudes towards mathematics (e.g., Friedman, 1994; Fennema & Hart, 1994; Halat, 2006). However, according to Hyde, Fennema & Lamon (1990) and Malpass, O'Neil & Hocevar (1999), there is also a considerable increase in the gender gap among gifted or high scoring students on mathematics tests. There are many factors, such as prior achievement, value, stereotyping mathematics as a male domain, and curriculum appearing to play vital roles in the sex differences between boys and girls in mathematics (e.g., Fennema & Sherman, 1978; Becker, 1981; Ethington, 1992; Grossman & Grossman, 1994; Forgasız, 2005; Leder, 2005; Halat, 2007).

As a conclusion, the study pointed out that the pre-service elementary school teachers' van Hiele levels were 0 (Pre-recognition), I (Visualization), II (Analysis), III (Ordering) and IV (Deduction). None of them showed level-V reasoning stages in geometry. Likewise, the pre-service secondary mathematics teachers' van Hiele levels were I (Visualization), II (Analysis), III (Ordering), IV (Deduction) and V (Rigor). Based on these results, it can be said that the pre-service elementary teachers' geometry knowledge is adequate to instruct at elementary school level, but it would be difficult to say the same thing for the pre-service secondary mathematics teachers who are expected to teach at high school level. Even though there was no significant difference between male and female pre-service elementary school teachers' van Hiele levels, there was a difference between male and female pre-service secondary mathematics teachers' van Hiele levels favoring males.

Limitations and Recomendations

The findings of the study cannot necessarily be generalized to other pre-service elementary school and secondary mathematics teachers because there are many factors, such as the academic abilities of students, quality of instruction, technological equipments, types of courses offered in the programs, and so forth appearing to influence students' mathematics learning that might be different at different institutions.

Moreover, a student can perform better in one area; yet not exhibit the same performance level in other areas (Fuys et al., 1988; Burger & Shaughnessy, 1986). Geometry topics investigated in this study were polygons. Therefore, the findings of the study can not necessarily be applied to all geometry topics, or other topics in mathematics.

I would like to finish by making some explicit recommendations for the teacher education programs. First, the mathematics teacher education programs should consider offering a Euclidean geometry course for the pre-service secondary mathematics teachers. Second, the teacher educators in the programs should give more support to female students in their classes.

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