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Developing New Views on Taken-for-Granted Assumptions: The Case of Division of Fractions

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Abstract

This article outlines a discussion had by a group of prospective teachers regarding questions that remained after their previous experiences as students. The discussion was focused on the rules associated with the division of decimals. Prospective teachers' initial discussion showed a strong tendency to handle the rules as taken-for-granted facts. As discussion progressed, however, they showed more accountability for their learning by connecting previously known concepts/explanations to justify the rules in the target question.

Keywords: Pre-service teacher education, Justifications, Reasoning

Introduction

Among many indicators of mathematical comprehension, the ability to justify is considered to be most critical as it is more important to be able to explain "*why* a particular mathematical statement is true or where a mathematical rule comes from" (Common Core State Standards for Mathematics [CCSSM] 2010, p.4) rather than simply executing the memorized rules. Teacher educators who teach elementary mathematics methods courses strive to help prospective teachers recognize the importance of mathematical justification and encourage the prospective teachers to experience the process. My K-8 mathematics methods course is not an exception in terms of its emphasis on mathematical justification. Despite this seemingly shared emphasis, however, we often face challenges and tensions regarding our approach. The first major obstacle is the prior learning experiences and knowledge of prospective teachers (e.g., Eisenhart et al., 1993; Thompson, 1992). This is an issue particularly for prospective teachers whose prior learning experiences relied primarily on what was taught by external mathematical authorities (e.g., teachers, textbooks), as it is a larger challenge to develop their own justifications and to reflect upon others' ideas (Cooney, Shealy, & Arvold, 1998; Mewborn, 1999; Wilson & Goldenberg, 1998; Wilson & Lloyd, 2000).

A second major obstacle is that it is practically impossible to discuss all K-8 content in one semester. In spite of this, prospective teachers expect to learn the mathematics information they should know as well as how to teach that information to students over that short time frame (Cooney, Shealy, & Arvold, 1998). This often creates feelings of insecurity among prospective teachers who are looking for teaching strategies they can easily implement in their immediate teaching settings.

In an effort to address these challenges, my course included a task that involves small group collaborative discussion and reflection on the questions students themselves raised. This article reports on one small group's discussion that highlights their critical stages of progress and also provides suggestions for continued growth.

Context: Discussion on Never Asked Questions

In the beginning of the semester, prospective teachers in my K-8 mathematics methods course were asked to look back on their past learning experiences as students and list several questions they had, but never asked. Each small group of four or five students then chose questions from the list to discuss with their group throughout the semester via weekly online discussion forums as well as selected in-class meetings. The group that I will discuss consisted of four members (Alex, Casey, Logan, and Shea, all pseudonyms) and the following was one of the questions they chose for their discussion:

> When dividing decimal numbers, we move decimal point to right to make the divisor a whole number and move decimal point in the dividend the same number of places. Why are we doing this?

Progress of Group Discussion and Reflection

Initial Discussion: Dependence on "What Someone Says"

This group initially focused on searching for resources that were published elsewhere (predominantly websites). At this stage, the discussion forum was primarily used as a repository of resources rather than an arena for mathematical discussion. Some evaluative comments were posted, but the focus of the evaluation was on the format or organization of websites rather than the mathematical content. However, when reviewing the provided web resources, they simply provided more *detailed steps* without further explanations. These steps do nothing to explain the act of moving decimal points and this process was not much different from the procedure stated in the original never asked question. This showed the prospective teachers' unfiltered acceptance of *what someone says*, especially when they believed that person had more expertise or authority on the subject. Although the open-ended nature of this discussion was emphasized before we began, there were also a number of attempts to get affirmations from me (the instructor) early on as students asked, "Is it right?" or "Are we on the right track?", which showed their perception of the instructor as a mathematical authority. This unfiltered dependency posed a major obstacle when trying to reveal students' own thoughts in the initial stage.

Shift 1: Emergence of "What-if" Questions

It took several weeks until Shea first stated, "I feel that none of the resources we found really explained why we need to locate the decimal point like that." Shea's comment prompted other students to critically examine the resources they had already found instead of searching for additional resources provided by someone else. Additionally, students started to generate questions that they had or questions that their potential students may have in the future. Three examples of such questions that triggered more concept-based discussions are as follows:

- Casey: "What if students don't know what they need to know? What should students know before they learn this procedure?"
- Logan: "I am worried about the misconception that young students may develop. We move around the location of the decimal point. If we do not make it clear what it means, children may think that the location of decimal point does not mean anything. What if my students ask me why we are not doing the same thing for the addition and subtraction? [e.g., 2.3 + 0.12 = 23 + 1.2 = 24.2: move the decimal point to right to make

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the first addend a whole number and move decimal point in the second addend the same number of places]"

- Alex: "Will it be wrong if we do the same thing when the divisor is a whole number but the dividend is not a whole number? [i.e., move the decimal point to the right to make the dividend a whole number and move decimal point in divisor the same number of places]"

These types of what-if questions created a shift that encouraged these prospective teachers to become absorbed in their own individual thoughts.

Shift 2: Relating Relevant Concepts to Address "What I think"

In this stage, prospective teachers started to search for relevant concepts to respond to the what-if questions they posed in a more active manner. Their attempts to extend the models or concepts they already knew became more visible through their efforts to explain the procedures of decimal division. Those included the concepts of place value, meaning of basic operations, equivalent fractions, and estimation.

Meaning of Basic Operations and Place Value. In response to Casey's question, group members initially agreed that the meaning of basic operations and the concept of place value should be the major prior knowledge. One sample excerpt in the forum is as follows:

> Shea: "When teaching the division of decimals, it is important for students to understand the concept of division before attempting to solve the problem…Students need to understand that division asks the question of how many groups of the divisor can be in the dividend. For example the division problem 2 ÷ 0.2 asks the question how many groups of 0.2 can be in 2."

When asked how the place value concept is related to decimal operations, students chose simple addition and subtraction questions to help model the process using base 10 blocks and drawings (Figure 1).

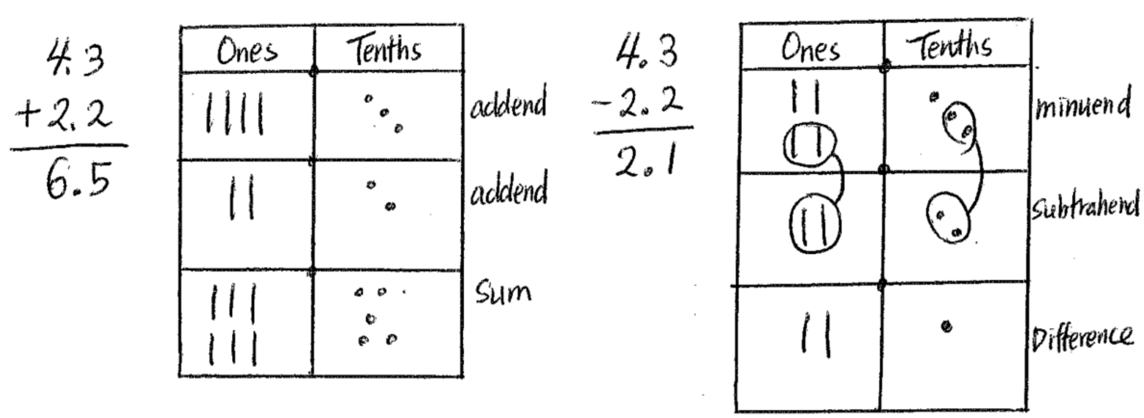

Figure 1. Drawings for addition and subtraction of decimals

These models are an extension of their discussion of algorithms for whole number operations that were done in class. During this process, the group responded to Logan's what-if question by using base 10 blocks to explain why the decimal point should not be placed in that way for addition and subtraction questions. This group believed that Logan's what-if question could also be resolved by clarifying the meaning of the addition/subtraction and place value concepts. In the case of multiplication and division, group members attempted to use the area model they used for whole number operations. They were able to demonstrate the process of division with simple problems, but it was not consistently successful to show the representation of the area model of division using base 10 blocks. I cannot describe in detail all the difficulties this group encountered due to the limited space of this paper. However, it was evident that the main issue was the confusion in uniformly labeling and interpreting the value of each piece of base 10 blocks that was used in the area model.

Estimation in Context. Alex suggested utilizing a money context to help students make sense of the process determining where to place the decimal point rather than focusing on the steps or rules:

> Alex: "It would probably be a good idea to use money-related contexts. Money notations contain decimal concepts. If I have $2.60 and a pencil costs $0.13, how many pencils can I buy? [2.60 ÷ 0.13] Obviously, we know that we can buy more than 2 pencils. We can easily estimate that the answer is 20. So, estimation will be a good aide to find a reasonable place for the decimal point in the quotient."

Other group members agree that this familiar money-related context would be helpful. However, they felt that this strategy alone could not explicitly explain the meaning of moving the decimal point. Rather, it can be used as a checking strategy to ensure that the answer found is reasonable.

Relating to Common Fractions. To provide a more generalizable explanation, students attended to the concept of fractions. Initially, however, group members expressed different opinions. For example, while Alex and Shea suggested looking at the relationship between common fractions and decimal fractions, Logan stated that it would be better to keep them separate because the associated solution steps/rules were so different. It was not that Logan ignored the importance of expressing decimals as fractions or fractions as decimals. However, she was afraid that the emphasis on this connection might hinder students from mastering the standard computation skill/procedure that was addressed in the original never asked question. In response to Logan's opinion, other group members highlighted the relationship between common fractions and decimals. The following post from Shea summarized multiple rounds of discussions on this matter:

> Shea: "Students need to fully understand the relationship between fractions and decimals before they begin to learn how to divide decimals…If there is an emphasis on the connection between the two, it might make grasping the idea of dividing decimals much easier, too. Therefore, I believe that teachers should teach these two forms together, not as isolated concepts."

Logan thought that common fractions were much easier for her in terms of visualization and she determined that this could also be true for young students. In this regard, she thought that representing decimal fractions as common fractions would help students develop decimal number sense. In the subsequent discussion, Logan explained the question by converting decimal notations into common fractions.

> Logan: "I could change the question to a fraction form (Figure 2, Step 1). I would explain to a student that it is simpler to work with whole numbers. Therefore, I would ask a student to find equivalent fractions by multiplying the same number to the numerator and denominator. We know that whatever we do to one side [divisor] of a division problem, we must do to the other [dividend] to keep the value of the fraction the same." (Figure 2, Step 2)

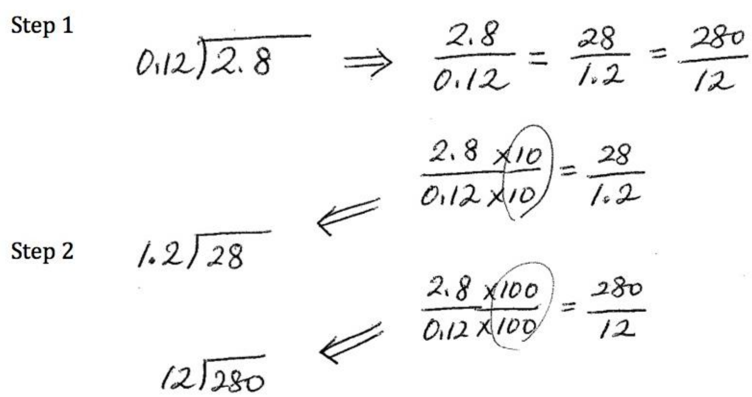

Figure 2. Using equivalent fractions

Group members agreed that Logan's explanation clearly justifies the action of moving the decimal point in the division process. Casey suggested looking at the what-if question that Alex posed earlier. Shea responded that it would still be mathematically correct, but it would not be necessary.

> Shea: "This process should make sense as long as we keep the fraction value the same but it is not needed to make unnecessarily big numbers. Not efficient." (e.g., Logan's example in Figure 2 can be continuously extended to 2800 ÷ 120 or 28000 ÷ 1200.]

This group concluded that relating the concept of equivalent fractions would be the most viable justification in terms of its efficiency and general applicability.

After Thoughts and Suggestions

This group's discussion revealed the potential found in using prospective teachers' *never asked questions* as a means to encourage their participation in a professional discussion. This context served as an invitation for the prospective elementary teachers, who are usually trained as generalists, to involve in more mathematically oriented conversations. Through this experience, prospective teachers engaged in multiple mathematical practices such as questioning taken-for-granted assumptions, detecting potential limitations of proposed ideas, predicting possible struggles young students may have, evaluating arguments, and making connections. The following section discusses the key elements the prospective teachers experienced and provides some suggestions for teacher educators who want to implement similar tasks for their classes.

Developing New Views on Taken-for-Granted Assumptions

When invited to form *never asked questions* from their previous education, prospective teachers revealed a variety of queries. Initially, the discussion on their own *never asked questions* was hindered by their tendency to rely on "what someone else says" instead of "what I think." Shea's comments showed her initial frustration: "At first, we just used a lot of outside resources rather than coming up with our own strategies. It was a challenge for me…to explain something that seems to be taken for granted by other people." Logan also stated, "This question required me to think deeper about the mathematical procedures I have not fully paid attention to before." These prospective teachers never asked this question in their previous experiences as students because they simply believed whatever their teacher or textbook (i.e., mathematical authority) taught them. However, when they continued to check if they had answered their own question (i.e., answering *why* we are doing in that way rather than *how* to do it), they found that there were not many resources available. The use of *never asked questions* provided these prospective teachers with a space to re-examine their own perceptions, knowledge, and skills so that they could show strengthened accountability for their own learning and plan for their future teaching. I would suggest that teacher educators continue to develop tasks that can connect prospective teachers' past experiences as students to their future experiences as teachers.

Thinking of "What-if" Situations

The what-if questions posed by prospective teachers encouraged their curiosity of the unknown and prompted them to explore reasons for the specific action in the procedure. While they tried to respond to their own what-if questions, they realized that the isolated rules would not be enough to justify the procedures at hand. Disagreement also served as a stimulus for new ideas and presented new opportunities to refine prospective teachers' existing understanding. For example, Logan was initially opposed to the interpretation of decimals as a form of common fractions because she believed this would cause more confusion for students. However, after multiple rounds of discussion, she developed a more organized explanation. In the small group discussion I reported here, what-if questions and disagreements were developed naturally. Teacher educators might consider including a more explicit mechanism that encourages prospective teachers to envision possible problematic scenarios.

Making Connections Visible

This discussion encouraged prospective teachers to connect several previously isolated concepts, such as the meanings of basic operations, equivalent fractions, and the role of estimation. During this process, they realized that the rules associated with dividing decimals were concrete, strategic, and relate directly to many other mathematical concepts. This experience confirmed the importance of making connections within mathematics through an in-depth exploration of fewer, yet still related, topics as supported by current mathematics education reform (CCSSM 2010; NCTM 2000).

I hope that this experience provided prospective teachers with an opportunity to tackle the seemingly taken-for-granted rules about which they have never inquired. I believe that this simple *never asked question* provided prospective students with ample opportunities to engage in a more personal exploration that helped them to make sense of the division algorithm involving decimals by constructing meaningful mathematical relationships. This type of approach may be an effective way to utilize the limited time given to teacher education programs. I also hope that the progress students made while tackling never asked questions could be a means to experience the standards for the mathematical processes we endorse by making sense of problems, constructing viable arguments, and critiquing the reasoning of others (CCSSM 2010; NCTM 2000). The opportunity to engage in these practices by themselves will ultimately be a more powerful experience for these prospective teachers.

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