Rectangles and Rhombi: How Well Do Preservice Teachers Know Them?

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Abstract

This descriptive study investigated preservice teachers' understanding of the properties and relationships among parallelograms. Forty preservice teachers in a pre-methods mathematics course for elementary education provided written descriptions of the terms rectangle and rhombus. These responses were sorted into categories according to similarities in description. Results show that only nine respondents articulated adequate descriptions of rectangle and only 1 respondent articulated an adequate description of rhombus. The findings are consistent with the body of literature showing teachers of young children are in need of further development of their conceptions of mathematics ideas - particularly in geometry.

Introduction

In discussing the impact of teacher knowledge on learning mathematics, Jones, Langrall, Thornton, and Nisbet (2002) observe that current models of teaching mathematics, including guided reinvention, Open-Approach Method, and the Mathematics Teaching Cycle (see Jones, et al, 2002 for references) all reflect the belief that instruction should be informed in part by teacher knowledge of mathematics. Perry and Docket (2002) also note that,

> One important point to make here is that if adults are to play the role of 'knowing assistant and supporter,' they need to know the mathematics with which their children are dealing. Not only do they need to be able to handle the questions posed, or at least be able to see a route toward a solution, but they also need to have what Ma (1999, p. 124) called a 'profound understanding of fundamental mathematics' (p. 103)

for which Perry and Dockett indicate few teachers of young children have.

Because the Geometry Standard from the National Council of Teachers of Mathematics *Principles and Standards for School Mathematics* states a desire for learners to "analyze characteristics and properties of two- and three- dimensional geometric shapes and develop mathematical arguments about geometric relationships" (NCTM, 2000, p. 41), teachers of elementary school children need sufficient geometry content knowledge to help them realize these curricular expectations for children's learning of geometry. The Conference Board of the Mathematical Sciences observes that much of the geometry content proposed in curricula is new (CBMS, 2001, p. 21). Furthermore, they advocate that teachers of young children develop competence in such content as, "Basic shapes, their properties, and relationships among them: developing an understanding of angles, transformations (reflections, rotations, and translations), congruence and similarity" (CBMS, 2001, p. 21) and in "Communicating geometric ideas: learning technical vocabulary and understanding the role of mathematical definition" (CBMS, 2001, p. 21). In short, a "profound understanding" of geometric ideas is needed by teachers. This suggests a need to examine teachers' understanding of geometry.

With regard to the CBMS recommendation that teachers be able to develop competence in properties of basic shapes and their relationships to each other, and to be able to develop competence in communicating geometry by developing vocabulary and "understanding the role of definition", one may inquire as to the development of this in preservice programs. Prompted by a study by Reinke (1997) who found that preservice teachers in an elementary education program presented evidence of confusion with regard to the concepts of area and perimeter, the question of the nature of other obstacles that geometry topics may present to preservice teachers is raised.

The present study examines what is revealed about preservice teachers understanding of the properties and relationships among parallelograms through their articulation of the meaning of rectangle and rhombus. Its purpose is to explore and diagnose any misconceptions these pre-service teachers have regarding these shapes.

Methodology

The participants in this study were 40 pre-service teachers taking the first course in a two-course sequence of mathematics content for teachers at a rural comprehensive university. The participants were randomly selected from four sections of the course taught by the same instructor. Geometry concepts were not a part of the curriculum for this course. Thirty-four of the participants had at least 3 years of high school mathematics. Six had fewer than 3 years of high school mathematics.

The study took place halfway through the semester. The first half of the semester was focused on the development of number and operation concepts. Each participant had completed at least one other college mathematics course that fulfilled a general core curriculum requirement.

The participants were presented with the following prompt.

> The purpose of this survey is to discover *your* perceptions of the meanings of the following mathematics terms. Please provide your personal definition for each of the following terms. Please use the back of this form if you need more room. For any terms that you do not know, simply write, "I don't know."
>
> 1) rectangle
> 2) rhombus

The participants were provided this prompt on a recording sheet that also requested their teaching concentration (minor), what college mathematics classes they have taken, and what high school mathematics they had. The participants were given as much time as they needed to complete their definitions.

Responses were analyzed for content and grouped according to similarities in description. Words, symbols, and pictures were all accepted in the respondents' attempts to define the terms. The term "rectangle" was considered "apparently correct" if the description provided excluded non-parallelograms and non-rectangular parallelograms, and included squares. The term "rhombus" was considered "apparently correct" if the description provided excluded non-parallelograms and non-equilateral parallelograms, and included squares.

Results and Discussion

Upon analysis of their definitions, only 9 of the 40, or 22.5%, provided a definition of rectangle that could be considered complete which include squares and exclude any parallelogram that did not have a right angle. Only 1 of the 40, or 2.5%, provided a definition of rhombus that would include squares and exclude any parallelogram that didn't have adjacent sides equal.

However, it is not the complete definitions that are most interesting, nor even how few of the participants got them correct, it is the misconceptions regarding these shapes that seem to be indicated by the other responses. Although 39 participants, or 98%, attempted a definition for rectangle, only 29 participants, or 73%, attempted a definition for rhombus.

For Rectangle, one participant's attempt was actually a definition of a triangle. The rest provided a range of responses. Table 1 presents these responses. Twenty-two responses (55%) for rectangle indicated that these people thought rectangles must have two sides longer than the other two sides. Of those, 16 failed to mention that rectangles must also have right angles. Two of the 40 (5%) acknowledged that rectangles must have four sides but did not acknowledge the criteria of right angles or parallelism. Two others (5%) did not acknowledge the criterion of four sides. Three of the 40 (7.5%) acknowledged the criterion of parallelism, but did not acknowledge the criterion of four sides.

With regard to the 22 participants who believed that rectangles must have two sides longer than the other two sides, it is rather interesting to note that U.S. culture does not provide a specific term for "non-square" rectangles that, for example, British culture refers to as "oblongs." This may contribute to the tendency of these participants to consider rectangles as "requiring" two sides be longer than the other two to distinguish them from squares despite the mathematical concept that rectangles encompass squares.

It should be further noted that these data do not suggest that these particular preservice teachers won't recognize rectangles when they encounter them, but, rather, present evidence that they are not articulating complete descriptions of them. What impact, if any, this could have on pupil learning is a question for further study.

For Rhombus, there was an even broader range of responses. Table 2 presents these responses. In addition to the one (2.5%) correct response, and the 11 (27.5%) "I don't know" or blank responses, there were 8 (20%) responses that identified the criterion that a rhombus must have four equal sides, but insisted that a square would not be included. Six (15%) participants suggested that a rhombus is a parallelogram, but that they did not contain any right angles, and that not all sides must be equal. Four (10%) participants stopped at identifying the criterion of rhombi being parallelograms. Four others (10%) defined them as trapezoids. Two (5%) suggested that they were a "type of square," whereas two others (5%) indicated only that they were quadrilaterals. Finally, there were two participants (5%) who incorrectly defined a rhombus - one as a "type of cube," another as "flat on one side and round on the other."

It is interesting to note that 14 of the preservice teachers emphasized rhombus as having a “tilt” or “slant.” This appears consistent with the tendency of early mathematics programs use of the word “diamond” to refer to such two-dimensional shapes. In this way a square that has been oriented to “stand on one corner” is considered a “diamond.”

Table 1
Participant Definitions for Rectangle

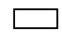
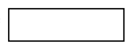
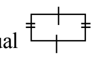
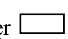
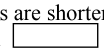
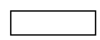
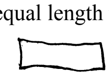
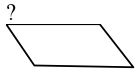
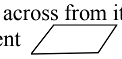
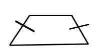
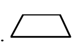
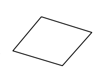
Type of Response	Definitions	N
apparently correct	• a geometric shape with 4 sides and 4 90□angles the sides opposite each other must have the same length • a 4 sided figure that has parallel sides equalling each other and all angles are right angles • 4 right angles and 2 pairs of sides equal in length (could be a square) • quadrilateral with 90□angles at each adjacent side with opposite sides equal • An object with four sides and four equal angles. Two sides parallel with each other are equal length. • A 4 sided figure with 4 right angles, the opposite sides are arallel, and all sodes do not have to be the same length • Four parallel sides, Four 90□<Õs, Perpendicular lines, with 2 pairs of lines  • a four-sided shape whoÕs opposite sides are parallel, with four 90□angles. • A rectangle in a shape consisting of four sides which form four right angles	9
rectangles must have four sides that are NOT all the same length (with reference to right angles)	• a shape that has four sides with 2 of the sides being one length and 2 of the sides being a different length with the interceptions of these sides being right angles • A shape with 4 sides- 2 shorter sides parallel to each other & 2 longer sides connecting them at 90□ angles • 4 sides, all 90□angles, two sides have the same length, the other two saides have the same length but are different from the first two. • a four sided shape with 2 sets of 2 equal sides on set exceeding the other in length, all angles = 90□ • A four sided figure with 4 90□angles, where the length does not equal the width • a longer looking square, with the two opposite sides equal	6
rectangles must have four sides that are NOT all the same length (with no mention of right angles)	• a shape that looks like a stretched square  • a box with two long sides and two shorter sides, each respectively to their own, are equal  • a polygon with 4 sides. The length always different than the width. Opposide sides = in length • 4 sided shape, with 2 diff lengths of sides being congruent • a shape with four sides, not equal sides • Is a four sided object. The length is of equal measurement and the widths have equal measurements • A rectangle is a 4 side object with either the height or the width is longer than the other • a shape which has 2 sets of equal sides which are different sizes • a shape where two sides are long & the other two sides are shorter  • Has 4 sides, 2 are longer than the other 2 shorter ones.  • A four sided shape with 2 sides that are longer than the other two  • A shape which has 2 po sets of parallel and equal sides. 2 sides are longer than the other two. • a four sided shape with two equal parellel lines on top and bottom, and to equal parellel lines on each side. All four sides are not equal • consists of 4 sides - 2 parallel lines longer than the width • larger than a square with 2 sides being equal to itÕs opposite • A rhombus with 4 sides, 1 pair of sides have equal length and is longer that the other 2 sides also of equal length to each other	16
no mention of right angles (or perpendicularity) or do not acknowledge parallelism	•  four sides • an object with 4 sides	2
no mention of 4 sides	• an object with opposite ends being parallel to each other 90□angles in each corner • in a rectangle the length is greater than the width	2
mention parallelism but are not restricted to rectangles	• a shape with 2 pairs of equal sides each pair being parallel • an object in which 2 parallel sides are the same length • A shape with parallel sides being equal in size but not the adjecent ones	3
No evidence of any understanding	• I don't know • A shape with three sides. There are 3 different types-obtuse, isosolese (sp?) & right.	2

Table 2
Participant Definitions for Rhombus

Type of Response	Definitions	n
Correct	• a parallelogram with 4 equal sides	1
"I don't know" or No response	• a drunken rectangle. I really donÕt know • I donÕt know • I donÕt know • I donÕt know • No response • I donÕt know • I donÕt know • I donÕt remember • I donÕt know • ? •  a shape?	11
Four equal sides but not a square (emphasis on "tilt" or "slant")	• a square that looks like it has been pushed over • looks like a slanted square with about 45□angles • a slanted square • an uneven square • Òsquished square,Ó4 equal sided shape different than a square in that the angles of a rhombus are not 90□ • a geometric figure that has 4 sides that are equal in length There are 2 pairs of parallel lines but doesnÕt have any right angles • A 4 sided figure with equal lengths, with 2 equal obtuse angles, and 2 equal angles smaller than 90□ • 4 sided figure with 4 equal sides, but two acute angles & 2 obtuse	8
Parallelogram without 90□angles and adjacent sides not equal (emphasis on "tilt" or "slant")	• A rhombus is a shapem that of like a slanted rectangle. Two sides strait, two sides diagonal (2 pair of 2 equal sides) • a tilted square with all sides not equal • A rectangle with two slanted sides • a rectangle with a slant • a rhombus is a rectangle tilted in one direction, opposite sides still parallel but angles in corners are now 60□or 120□ • Like a rectangle, but 2 of the sides are at angles other than 90□.	6
General Parallelogram	• a geometric shape with 4 sides. each side must be equal in length to the side across from it • quadrilateral with opp. angles being congruent. and opp. sides being concruent  • a rhombus is a 4 sided object consisting of 2 sets of parallel lines • A four-sided shape whose opposite sides are parallel but donÕt necessarily meet at 90□angles	4
Trapezoid	• a shape in which only 2 sides are =  • the top is short, & the sides and bottom are longer.  • An object with 4 sides and 4 angles. Two sides are parallel to each other ae equal and two angles across from each other are equal • 4 sides, 2 sides || with the other	4
Type of Square	• type of square • A diamond shaped square where the sides are parallel 	2
General Quadrilateral	• A quadrilateral which the sides are at angles • A four sided figure	2
Other Incorrect	• This is a type of cube • A rhombus is flat on one side and round on another	2

Implications

A study of the frequency of occurrence of mathematics vocabulary in elementary school mathematics textbooks (Pickreign, 1996) found that the term rhombus occurred 70 times and "diamond" occurred 20 times out of a total corpus of 480,977 mathematics terms and phrases. Given the potential "lack of exposure" in curricular material, one might find preservice teachers uncertainty regarding rhombus as unsurprising. Yet, as with the uncertainty evidenced in the description of rectangles, this raises the question of these participants' experiences with these concepts in middle or high school.

Regardless of their experiences with the concept, the results presented in this study are that most of these participants lack the ability to articulate complete descriptions of rectangle and rhombus. This may very well be evidence that most of these participants are operating at the Visualization level (level 0) of the Van Hiele theory of geometric cognition (Van de Walle, 2004). That is, "the products of thought at level 0 are classes or groupings of shapes that seem to be 'alike'" (Van de Walle, 2004, p. 347). Consequently, the participants who are willing to insist that rectangles are "longer than they are wide" have decided that is what rectangles look like. Likewise, those participants that described rhombi as requiring a "tilt" or "slant" have decided that is what rhombi look like.

This presents significant implications for teacher education. One of the characteristics of the Van Hiele levels is that "geometric experience is the greatest single factor influencing advancement through the levels" (Van de Walle, 2004, p. 348). Hence, it is conceivable that these participants lack sufficient experience with these geometric ideas to have advanced to a level of geometric cognition permitting them to completely describe rectangle and rhombus. If this is so, then where are these people who are pursuing teaching licensure to obtain the necessary experience with these concepts? The obvious place, for these people, is somewhere in their teacher education program.

This, however, raises serious questions to be answered if we are to affect the content knowledge of teachers of mathematics: Can sufficient experience with these geometric ideas be provided in teacher education programs to lead to more profound understandings? Are there other mathematical ideas that require such experience? What should characterize these experiences? Can these experiences be provided without adding time or credit hours to teacher certification programs? One option in addressing these questions might be to not address them. Instead, accept teacher education programs as entry level preparation and utilize continuing professional development to address the growth of teachers' profound understanding of fundamental mathematics.

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