From Whole Class to Small Groups Instruction:

Learners Developing Mathematical Concepts

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Abstract

This case study examines the ways in which a teacher education program develops prospective teachers who implement the principles of constructing mathematical knowledge via a learning discourse. The study presents and analyzes a mathematical discourse in a 4th grade class as well as the feedback conversation of prospective teachers with their methods supervisor. Findings suggest that there are at least five main components teachers have to relate to in order to bring about change in the pupils' discourse. The article concludes with a discussion of the need to build a different culture in schools and in teacher education.

Introduction

This article presents a case study which focuses on mediation through mathematical learning discourse in elementary school classes by classroom teachers, prospective teachers and pupils. The challenge to mathematics teachers is particularly complex and requires multiple kinds of knowledge as well as coping with tensions and contradictions between stability and flexibility, decomposing practice into small units and maintaining the integrity of the discipline. Much of mathematics teaching is still instrumental and focuses on technical calculations and test results. Thus, in spite of the advancement in the research, there is not enough detailed knowledge of how to practically meet these challenges and dilemmas within the mathematics classroom.

Mathematical Discourse

There are three kinds of knowledge that are interwoven in teaching across most disciplines: (1) content knowledge, which encompasses deep understanding of the big ideas of the discipline as well as its scientific concepts, processes and the relationships among them; (2) pedagogical knowledge, which includes knowledge about the curriculum and core tasks of teaching, mainly, competence to translate theoretical knowledge into practical knowledge, scaffolding and demonstrating them for students; (3) student knowledge which means mainly sense-making, identifying students' mistakes and misconceptions and taking them up in order to leverage their thinking. However, the emphasis of ambitious teaching lies in the teacher's ability to adjust both content and pedagogical methods to students' prior knowledge, thinking and performance during interactive teaching (Grossman & McDonald, 2008; Hiebert & Morris, 2009; Lampert, & Graziani, 2009).

Relating specifically to mathematics, one of the key ways of effective teaching is orchestrating mathematical discourse in class. Conducting learning discourse requires the interweaving of all three kinds of knowledge in order to facilitate students' construction of knowledge and thinking as well as connecting them to mathematical big ideas. Stein, Engle, Smith, & Hughes (2008) suggest a five practices model of facilitating mathematical discussion: (a) anticipating likely student responses and preparing responses to them, (b) monitoring students' responses (c) selecting particular students to present their mathematical responses (d) sequencing the responses, (e) making connections between student responses and the key mathematical ideas.

Ball, Sleep, Boerst, & Bass (2009) also emphasize some of the practices mentioned in the model and articulate 4 main features of a productive whole-class math discussion: (a) the task chosen has to be accessible to students and lend itself to discussion. (b) initiation of the discussion has to afford quick understanding and take up by students. (c) the discussion has to be varied and inclusive and maintain a mathematical focus. (d) the conclusion of the discussion has to be connected to aspects of math content and process and should reinforce the productive norms of math discussion.

While Stein et al. (2008) and Ball et al. (2009) describe models and features of a mathematical discussion as a key practice and a means of teaching mathematics, Sfard (2007) regards the discourse as the very object of learning, and relates to learning mathematics as a change of discourse.

The Commognitive Framework

Ana Sfard (2007, 2008) views mathematics as a form of communication and rejects the split between thinking and communication. In order to stress this unity, she combines the terms cognitive and communication into a new adjective commognitive. Thus, learning mathematics according to this approach is modifying the learner's mathematical discourse.

Sfard (2007) relates to words and their use as one of four main interrelated features that make the discourse distinct. The other three features of the discourse are: visual mediators; mathematical narratives and routines. Transformations in each of these features are indications of the development and knowledge of the learners (Sfard, 2007).

A powerful strategy for learning according to this approach is creating a commognitive conflict that will trigger a sense of inconsistency in the learner regarding his understanding of the mathematical subject, and will induce him to recognize the need for change. The meta-level learning is most likely to originate when the learners are exposed to different ways of thinking and strategies and compare and contrast them to their original thought patterns. During such a discourse, the learners make sense of their and their peers' thinking and thus figure out the inner logic of the discourse (Sfard, 2007). Thus, the role of the teacher as a mediator is central mainly in conducting the discourse and stimulating the commognitive conflicts.

The broad aim of this case study is to examine, in the light of the literature, the ways in which our teacher education program develops prospective and veteran teachers who implement the principles of constructing mathematical knowledge via a learning discourse.

Context and Participants

The case study is situated within an experimental innovative teacher education program in the context of partnership between our College and one of the cooperating elementary schools (Margolin, 2007). The case described and analyzed in this article is an example taken from one of our partnership schools, specifically focusing on mathematical instruction in 4th grade. The focus in this article is on mathematical discourse. Specifically, we discuss the dramatic change in mathematical discourse from traditional frontal instruction to a constructivist open discourse in which knowledge is socially constructed.

Methods and Data Sources

We used a case study methodology because it is an empirical inquiry that investigates a phenomenon within its real-life context and it enabled us to conduct an in-depth investigation of a single event in an environment where the boundaries between the specific lesson and the context are not clearly evident (Yin, 2003).

Multiple sources of evidence was collected for one academic year: (1) 150 transcripts of prospective teachers' observations in mathematics discourse in 4th grade, taught by their peers, by the teachers or by the methods supervisor. (2) transcripts of audiotapes of 30 community meetings of the methods course. 3) final action researches of the 18 prospective and in-service teachers.

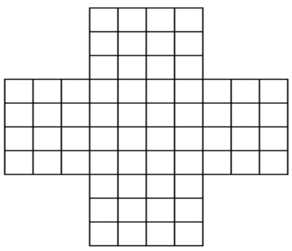
Through dialectical process between the data and the literature we defined 4 reoccurring main characteristics of the discourse: a) the challenging and open learning task b) the use of scientific mathematical accurate language c) the use of appropriate visual mediators d) routines such as questions, explanations, argumentations and commognitive conflict as inherent parts of the discourse. In order to validate these findings we read the methods course transcripts and the prospective teacher' and teachers' studies and identified their main strategies: a) considerations in selecting and articulating the learning tasks b) mediation via questions of clarification and explication instead of transferring information; c) considerations in selecting and using visual mediators; e) stimulating commognitive conflicts f) bootstrapping via all these means between intuitive concepts and scientific concepts and ideas. Finally, we selected one specific case of a lesson and of the teachers' discourse following it which represent the main findings.

Findings

We present and analyze an example of a discourse which took place in a 4th grade class of 35 pupils and characterizes the kinds of mathematical discourses that often occur in the elementary school described above. The pupils work in small groups of about 6 pupils.

At the beginning of the lesson a shape is projected on the board (see figure 1).

Figure 1: Shape presented by the teacher



The teacher asks the pupils to find the area of the shape, each one in his own way and emphasizes that they have to be able to present and explain their solution and the way they got it. The mentor teacher (Soly) conducts the discourse with a group of 6 pupils

(Arad, Nir, Tami, Tal, Neta, Roy), while the rest of the class explores and discusses the problem in their groups.

The Selected Instructional Task

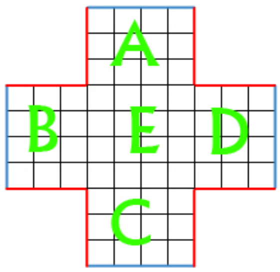
The task chosen affords a situation in which there are mathematical concepts and big ideas. Furthermore, it is open-ended, lends itself to discourse and has a potential for exposing and presenting possible strategies for solving it. It has also a well defined mathematical goal of understanding the meaning of area of compound shapes and differentiating it from perimeter. It is accessible to all pupils in the group as the teacher is acquainted with their prior knowledge and is aware of their ZPD - zone of proximal development (Vygotstky, 1986).

During the pupils' exploration of the task, the teacher planned the sequence of their responses and turned to Arad:

Soly: Now tell me please how you calculated the area of the shape.

Arad: I calculated the area of this cube (points on rectangle A in figure 2)

Figure 2: First and third response



Soly: Do you mean the area of the rectangle?

Arad: Yes, and I multiplied 4 times 3, because there are 4 like this, and I added the middle part which is 4 times 4.

Soly: What do you think (turns to the pupils)?

Nir: I did not understand

Soly: Who can explain what Arad did?

Nir: Aha, (looks at the shape) it is correct, I already understand why 4 times.

Soly: How can you write it in Mathematics (turns to Nir)?

Nir dictates (including brackets) and the teacher writes down: 4X(4X3)+4X4

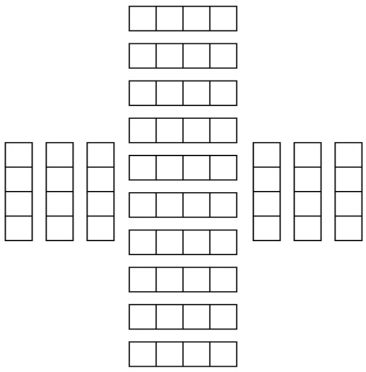
Soly: Do you need brackets?

Nir: No, because multiplication precedes addition and in multiplication we solve as it is written.

As a starting point the teacher selected Arad's right strategy and asked him to present and explain it to his peers. Arad uses the drawing of the shape in order to explain his way, but calls the "rectangle" a "cube". The teacher reminds the pupils to use the right mathematical concepts and while pointing on the rectangle in the shape she emphasizes the name of it by asking Arad: Do you mean the area of the rectangle? Then she asks the pupils what they think about Arad's response. Nir did not understand Arad's strategy, but the teacher delayed her response in order to enable him to look at the shape again and reconsider Arad's explanation. He catches it by himself. In order to be sure that he understood, and to demonstrate the answer to the pupils, she uses another visual mediator; she asks him to think about a number sentence. She takes advantage of his sentence and by asking about the brackets she causes Nir to repeat one of the rules of the order of operations before turning to the next pupil, Tal.

Tal: I counted how many squares are there in a row and added them 4 and 4 and 4…… (see figure 3)

Figure 3: Second response



Soly writes down: 4+4+4+4……+4 and asks Tal: like this?

Tal: Yes

Soly: What do you think?

Neta: It is also correct, but it is a lot of times 4

Soly: So, is it correct or incorrect?

Arad: It is similar to my answer, but I multiplied each 4 and she counted them

The second strategy was simple and the teacher demonstrated it by a number sentence. One of the pupils indicated its disadvantage.

After these two strategies the teacher invited Nir to present his wrong way:

Nir: I multiplied 8 times 3 (points to the 8 red lines in figure 2) and added 4 times 4 (the blue lines)

Soly: One minute, I am writing it down: 8X3+4X4. What do you think?

Tal: One minute, it is wrong, it can't be 64

Soly: We talk about various ways of finding the area and you say that this is wrong, but maybe there is a mistake in the calculation and the way is right? What do you think?

The teacher tried to stimulate a commognitive conflict between the two concepts Nir had confused – area and perimeter – by insisting upon the way, though the sum was incorrect.

Arad: It is not the area, it is around

Soly: What does it mean "around"?

Nir: It is the perimeter.

Tami: I don't understand anything

Soly: Who can explain what the problem is? What is the difference between Arad's way and Nir's way?

Arad: Look, he combined 8 times this line (the short sides of the shape) and added 4 times this line (the long sides) and this is not the area.

Soly: What do you call this line in the shape?

Neta: A side

Soly: How many sides do we have in the shape?

Pupils: 8, 12, no, 12.

Soly: Do you mean that he added 8 times the short side and 4 times the long side (the teacher points on the sides)?

Nir: Yes, I said that he calculated the perimeter (points with his finger to the lines of the perimeter)

Soly (turns to Tami): Are you with us? Please sum up the difference between the first suggestion and the second one (points to the 2 number sentences).

Tami: Arad found the area of each rectangle and combined all of them and Nir combined the sides.

Arad: He found the perimeter.

Dealing With Incorrect Answers

The teacher decided to present the wrong response after the two right ones and exposed a common confusion between the two concepts in order to hone in on the distinction between perimeter and an area. At face value, the strategy looks correct and similar to the first answer, but the first was multiplication of one side of the rectangle with the other and in this answer the side was multiplied with the numbers of times it appeared.

She also used the visual mediators and asked the pupils to explain their argumentation by demonstrating it on the drawing. When a pupil does not understand a response, the teacher leads him to find answer out by himself. In this case the drawing served as scaffolding for the correct response.

Using Visual Mediators on the Basis of Anticipated Responses

Tami: I counted the squares and got 63

Soly: What do you think?

Tal: She calculated

Soly: What do you think?

Nir: Yes, but……

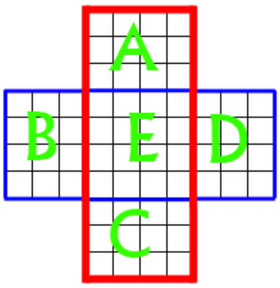
Tal: Yes, but it is also possible

Soly: This is also a legitimate way, but we can miscount, check again if it is 63

Here the teacher confirms the solution but warns from the possible mistake of miscounting.

Neta: I have an additional way.

Figure 4: Last response



I calculated the area of this rectangle (points on the horizontal blue rectangle in figure 4), 10 multiple 4 and this rectangle (points on the vertical red rectangle), and added them.

Soly: Do you mean 10 multiplied by 4 and another 10 multiplied by 4? What do you think?

Nir: It is not so correct

Soly: Can it be in math "not so correct"?

Nir: So, it is not correct because she counted the middle twice. It has to be only one big rectangle, 10 times 4 and 4 times 3 and another 4 times 3

Soly (turns to Tami) what do you think?

Tami: I don't know.

Soly: Try to dictate the appropriate sentence for me (the teacher colors in the areas Neta has mentioned)

Tami (counts the rows,) 10 times 4

Soly: And what else is left?

Tami: 4 times 3 and another 4 times 3

Soly: So (writes down): 10X4+3X4+3X4. Is it possible to write it briefly?

Pupils: What do you mean?

Soly: Is it possible to write this number sentence in a shorter way?

Tami: Yes, 10 times 4 and 3 times 4 times 2: 10X4+3X4x2

Nir: You need brackets

Pupils: You don't need them

Soly: Do you need them or not?

Pupils: It is multiplication. We do it according to the written order.

The embedded misconception in Neta's response is related to the misunderstanding of the idea that area of a shape is the sum of its parts. Again, the teacher mediated the concept of area by leading another pupil to explain the mistake and conclude the correct strategy. Nir discerned that Neta calculated the middle square in the shape twice. The teacher again stressed the endorsed mathematical narrative and asked: Can it be in math "not so correct"? Thus she reminds the pupils that in mathematics it can be either correct or wrong. She also reminds them the rule of order of operations by asking about the use of the brackets and letting a pupil to repeat it aloud. However, the teacher did not discuss explicitly the wrong answer and did not emphasize the double calculation of the middle square.

The lesson ends in summing it up:

Soly: Who wants to sum up our lesson?

Roy: We saw that there are many ways to calculate an area

Soly Projects the original shape and other compositions of the shape that represent the responses of the pupils (figure 5).

These are the ways you used to find the area of the shape. What are the differences and similarities among them?

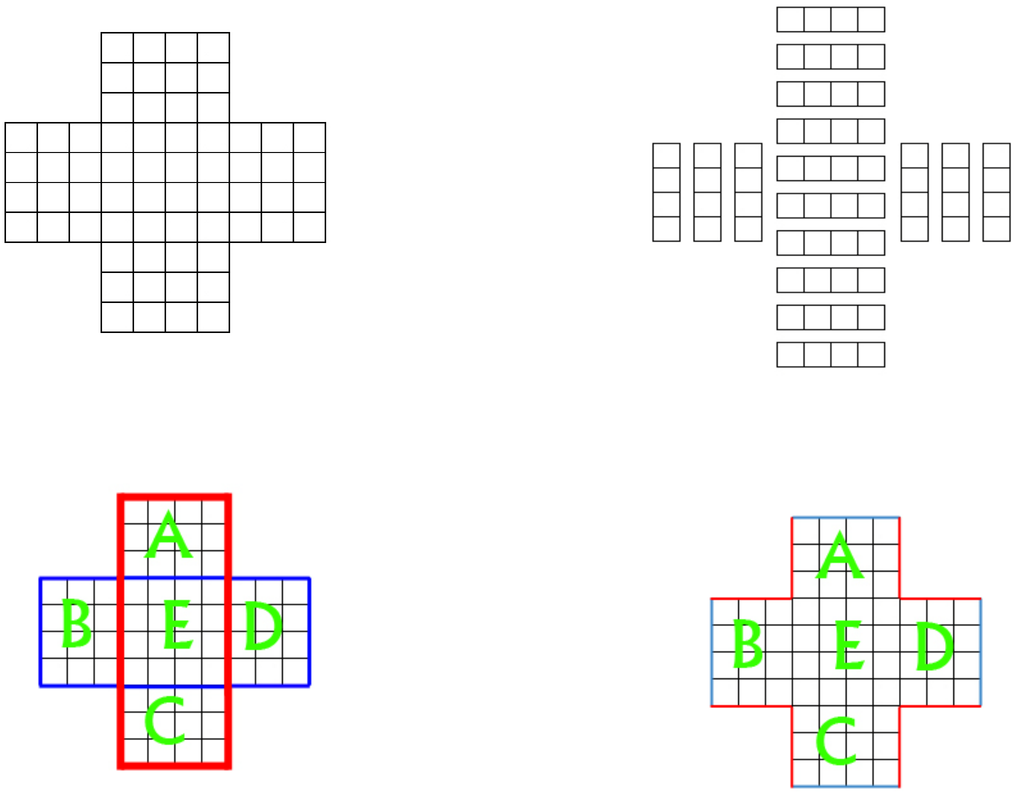
Arad: All of us had the same shape

Tal: Each of us solved it in a different way

Soly: How is it different, and why is it correct?

Nir: Each of us decomposed the shape into different parts in a different way

Figure 5: All figures



Roy: But together, it is the same shape, no matter how you decompose it as long as you don't do the same thing twice.

Soly: The conclusion is that in order to find area of a shape it is possible to decompose the shape into parts in different ways and then the area of the shape is the sum of all its parts.

Next lesson we will start from this point: I'll present the four figures that represent the various strategies and we shall try to distinguish between them and see which of them are more or less effective.

The teacher had no time to hear the ideas and concepts of the lesson from the pupils in their own words in order to evaluate their understanding, which she often does; thus she decided to come back to it next lesson.

The Feedback Conversation

After each lesson there is a feedback conversation that is devoted to analyzing the lesson observed on the basis of the students' documentation. The following part sums up the supervisor's main remarks relating to the strategies the teacher used:

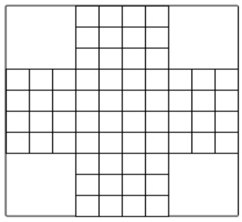
> The main idea in this lesson was that it is possible to find the area of a shape by decomposing and recomposing it in different ways without changing the area. However, a specific engagement in the concept of area which is one of the big ideas of the discipline was missing. The pupils tried to combine various shapes intuitively, without understanding the mathematical ideas of finding the area. The transition from intuitive answers of combining the shapes to a **mathematical narrative** of finding the area and not only calculating it had not yet been achieved.
>
> The **main concepts** in this discourse are: shape, area, perimeter, rectangle and side. These are concepts that the pupils have already learned, but it does not mean they know to use them correctly. Thus, we have to use each opportunity in order to continue and construct the meaning by **bootstrapping** between the scientific concepts and the pupils' intuitive concepts. The bootstrapping occurred in a minor way during the lesson though the task afforded deep discourse on the scientific concepts by enabling diverse ways to solve the problem, but that was underutilized.
>
> The teacher wanted to be sure by means of explicating the responses that the pupils understand the meaning of the concept. On this basis she stimulated a **commognitive conflict** by presenting an incorrect strategy through which she clarified the 2 concepts: area and perimeter.
>
> The use of the two **visual mediators** served as means to construct the scientific concepts. The drawings of the shape and its parts were very important and promoted the discourse. The pupils used them in order to show or prove their argumentations and demonstrate their solutions, and the teacher used them in order to demonstrate, explain or bootstrap between the pupils' intuitive concepts and the scientific ones. As the teacher anticipated the various solutions both correct and incorrect that the pupils might use, she prepared the visual mediators. She designed the whole as well as the decomposed shapes of each anticipated response in order to facilitate the pupils' understanding and in order to compare them. Thus, the various drawings demonstrated very clearly the main idea of the lesson, that you can find the area of compound shapes in many ways by decomposing them into sub-shapes, the pupils had already learned. The second visual mediator, the number sentences is another important visual mediator which can verify the pupils' understanding and demonstrate a mathematical narrative.

As one of the significant aims of teaching mathematics is to enrich the pupils' repertoire of strategies, the supervisor wanted to present an additional effective strategy. Thus, she raised the question if it is worthwhile introducing to the pupils a strategy that no one thought about. She presented the shape surrounded by a square (figure 6).

In this shape one has to calculate the area of the square (10X10) and then subtract the area of the four squares created in the corners 10X10-4(3X3).

The decision to introduce an additional important strategy that no one has used depends on the context of the lesson, namely on the time given, the focusing of the aim, the zone of proximal development of the pupils etc. Moreover, the conception of this strategy is different from all other solutions because it demonstrates a possibility the pupils didn't think about: to decompose the shape and recompose its parts into another shape (in this case a square) whose area they already know how to calculate, instead of decomposing it to sub-shapes they already know to calculate. However, if the aim is finding the area and not enriching the strategies may be it is needless presenting it.

Figure 6: Additional Strategy



Discussion

The premise of this article is that teaching mathematics is not only the acquisition of a collection of procedures and formulas but a transformation of thinking and language through mathematical discourse. We claim that in order to achieve the aim of changing the pupils' discourse, in order to stimulate meaningful teaching and learning and in order to prepare prospective teachers for these tasks some necessary as well as sufficient conditions should exist in schools in general and in classes in particular.

Small Groups Inside Class

A meaningful mathematical discourse in which the teacher can observe each pupil's engagement in the task, identify his zone of proximal development as well as misconceptions and relate to them in order to afford construction of concepts and ideas, can occur in small groups. In a whole class discussion only few pupils have the opportunity to articulate their thoughts or to expose their misconceptions publicly and the teacher can't really know about the others' understanding or relate to their difficulties.

This was the main and most dramatic change in the school's culture: Instead of the traditional question-and-answer "ping pong", the teachers learned to allow time for thinking and not to expect the pupils to answer correctly immediately; they turned the pupils into real partners in the discourse, communicating, responding to their peers and exposing their difficulties. While the teacher works with one group the rest of the class inquire or drill individually or in small groups. As the discourse is relatively intensive and short, the teacher can work with more than one group per lesson. The socio-mathematical norms and the routines that support a collaborative inquiry and group work among the young pupils and the various adults who co-teach inside class, established a culture that is different from the one that is prevalent in traditional frontal lessons.

Strategies of Mediation

Beyond the teachers' mathematical knowledge and the knowledge of their pupils, and on the basis of the environment described, they have to know how to mediate the mathematical concepts and big ideas to their pupils. We identified five main strategies the teachers learned in order to lead a meaningful mathematical discourse:

1. The learning task was open, challenging within the zone of proximal development of the pupils, and enabled multiple ways of thinking and afforded a meaningful discourse.

2. The teachers insisted upon using a scientific mathematical accurate language and bootstrapped between the intuitive concepts of the pupils and the scientific mathematical concepts and ideas of the discipline.

3. They anticipated the probable misconceptions and mistakes in each subject and encouraged their exposure. The teachers used these misconceptions as a means to leverage the discourse in order to align the pupils' strategies to the mathematical knowledge and facilitate the construction of correct responses.

4. The use of appropriate visual mediators was critical in order to facilitate the understanding and demonstrate the concepts and ideas. In our example the teacher used the drawings of the shape and the various ways the pupils proposed to find its area. She also used the number sentences in order to clarify or verify the pupils' understanding.

5. The routines, such as questions, explanations, argumentations and commognitive conflict were inherent in the discourse. The main repetitive pattern in the discourse above is the teacher's responses to the pupils' strategies. She almost always asks either the responder or the whole group rather than explaining. She wants the participants to think about the presented strategy, to confront it with theirs, confirm their understanding or expose their misconceptions. The teachers' responses invite clarifications, verification, accuracy of concepts, or questions that expose consent or non acceptance in order to stimulate conflicts.

Implications for Teacher Education

In the light of all this, it seems evident that transforming the prospective teachers' education is not sufficient; it is also necessary to establish concurrently a school culture that nurtures thinking and facilitates constructivist discourse among learners. Such a culture can be created through partnerships between colleges and schools, resulting in communities of practice that afford sustainable discourse among academic faculty, teachers, prospective teachers and pupils. Only by remaining updated, engaging in lifelong learning, garnering experience and gaining insights through the dialectic between theory and practice at all levels can teachers achieve meaningful learning. Prospective teachers are educated in this culture from their first day in the college and understand that this is their first phase of their long life professional development. The outcomes of the study imply a need to build a different culture in schools and in teacher education – one that enhances thinking and creates a mathematical discourse among learners.

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