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The Impact of Analyzing Student Work on Preservice Teachers' Content Knowledge and Beliefs about Effective Mathematics Teaching

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Abstract

This study investigated the impact of analyzing student work on preservice teachers' knowledge and beliefs about mathematics teaching. Forty-two prospective teachers participated. Data were collected using established instruments from the Integrating Mathematics and Pedagogy (IMAP) project and from the Learning Mathematics for Teaching (LMT) project. Results are shared along with the protocol used to guide the treatment. Findings show that preservice teachers who learn through the analysis of student work experienced a significant impact on their beliefs regarding how mathematics should be taught. Findings also show student work analyses did not diminish content knowledge development. Implications for undergraduate mathematics education courses are presented.

Keywords: *mathematics; education; preservice teachers; content knowledge; beliefs*

Introduction

The need for increasing mathematical competencies among our citizens has been a point of focus over the past few decades (e.g., California Space Education and Workforce Institute, 2008; Gardner, 1983; NCATE, 2010). An identified lack of mathematical literacy in the United States has been a major factor driving this focus. For example, Phillips (2007) reported that high numbers of adults struggled with daily tasks involving mathematics, including computing interest paid on a loan (78% of those involved), calculating miles per gallon when traveling (71%), and determining a 10% gratuity for a lunch bill (58%). These deficiencies are due, at least in part, to the mathematics education they received during their days as primary and secondary students.

Despite these alarming percentages, students can and should learn mathematics in deep, conceptual ways that lead to mathematical literacy (NCTM, 2000), which has been called the new literacy necessary for success in the world (Friedman, 2005; Schoenfeld, 1995). And, there is no greater impact on students' mathematical learning than that of a knowledgeable teacher (Hill, Rowan, & Ball, 2005; Rowan, Correnti, & Miller, 2002; Wiliam, 2014). If we hope to improve the mathematical literacy of our students, we must focus on the knowledge base teachers bring with them to the classroom.

The Conference Board of Mathematical Sciences (CBMS) (2012) has provided two recommendations for the knowledge preparation of preservice teachers (PSTs): (1) PSTs need mathematics courses that develop a good understanding of the mathematics they will teach, and (2) coursework that allows time to engage in reasoning, explaining, and making sense of the mathematics they will teach. Ultimately, PSTs need university courses that develop these skills and understandings in order to prevent them from relying on their past experiences as learners of mathematics that likely did not focus on reasoning, explaining, and sense making (CBMS, 2012).

However, defining specific ways to adhere to these recommendations is a complex task. How can we best develop and promote reasoning, explaining, and sense-making in an ongoing effort to prepare PSTs to teach mathematics effectively to their students?

Knowledge for Effective Mathematics Teaching

Several studies have provided grounding for the existence, conceptualization, and assessment of a robust teacher knowledge base necessary to support effective mathematics teaching (e.g., Ball, Thames, & Phelps, 2008; Carpenter et al., 1989; Cobb et al., 1991; Hill, Ball, & Schilling, 2008; Hill, Rowan, & Ball, 2005; Saxe et al., 2001; Shulman, 1986a; Shulman, 1986b; Shulman, 1987). This knowledge base is known as mathematical knowledge for teaching (MKT) (Hill, Ball, & Schilling, 2008, p. 377), and it works to divide the necessary knowledge into either subject matter knowledge or pedagogical content knowledge (Figure 1). Subject matter knowledge refers to various ways of understanding mathematical content that includes knowledge that is independent of teaching and learning contexts (common content knowledge), knowledge needed to follow students' mathematical thinking (specialized content knowledge), and knowledge of how mathematical topics build sequentially on one another (horizon content knowledge). Furthermore, pedagogical content knowledge refers to various ways of understanding the interconnection of mathematics and teaching that includes knowledge about how students interact with mathematical content (knowledge of content and students), knowledge about how topics and examples should be sequenced (knowledge of content and teaching), and knowledge of how mathematical topics fit into the larger body of mathematics (knowledge of curriculum).

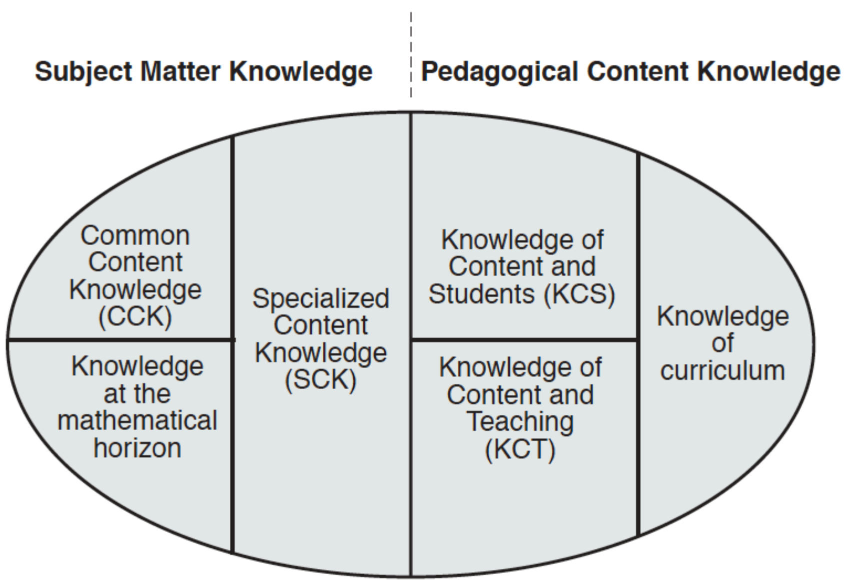

Figure 1
Mathematical knowledge for teaching (MKT).

According to this conceptualization, PSTs need to develop these types of knowledge specifically in order to teach mathematics effectively, i.e., to help students develop a deep, conceptual understanding of the subject. But, as is typical in educational practice, no single element occurs in isolation. A focus on MKT development alone is insufficient in helping PSTs

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prepare to teach mathematics effectively. The beliefs that PSTs hold about mathematics and learning mathematics must also be addressed.

Teacher Beliefs

Research has shown that a focus on developing knowledge for PSTs without simultaneously focusing on their beliefs is counter-productive (e.g., Ambrose, 2004; Philipp et al., 2007; Sowder, 2007). Beliefs have strong impacts on how PSTs learn mathematical content by acting to filter out what is deemed unimportant. For example, if PSTs learned mathematics in absence of reasoning, explaining, and sense-making, it is highly possible they believe that to be how mathematics should be taught.

Defining Beliefs. Beliefs are psychologically held understandings, premises, or propositions about the world that are thought to be true – they are lenses through which we see the world, dispositions towards our actions, and are held to varying degrees of conviction (Philipp, 2008). Ambrose (2004) found that PSTs' beliefs affected the way they taught as well as what subject matter they felt comfortable teaching and deemed worthwhile. Since beliefs have this kind of impact, it is important to identify and develop those that align with MKT and effective mathematics teaching.

Specifically, Philipp et al. (2007) defined the following as the seven beliefs necessary for effective mathematics teaching (see table 1 below). It is important that prospective teachers are afforded opportunities to develop both the mathematical knowledge (i.e. MKT) and beliefs outlined here if they are to be prepared to teach mathematics in effective ways.

Developing Knowledge and Beliefs

Research has also shown that developing teacher knowledge and beliefs within the context of K-12 educational settings is effective (Darling-Hammond and Baratz-Snowden, 2007). There are many factors that influence PSTs' beliefs and knowledge for teaching mathematics, however, using student work has shown considerable promise (Crespo, 2000; Kazemi & Franke, 2004; Son & Crespo, 2009). More specifically, the use of student work in these studies typically consists of examining solutions for correctness while also attempting to make sense of solution strategies, calculations, and approaches in an attempt to make claims about the students' levels of understanding. Ultimately, the goal is to make decisions about future instruction based on current levels of understanding. This is the operational definition of "student work analysis" utilized for this study.

Analyzing students' work in this manner has been extensively endorsed for teacher development, including its appearance as one of the central tasks of mathematics teaching (NCTM, 1991). For example, Slavit and Nelson (2010) found that asking teachers to examine their students' work for understanding sparked conversations and analyses that led to theory development regarding how future teaching might change to improve students' mathematical understanding. Other studies have examined the use of student work specifically in the development of various elements of MKT (e.g., Crespo, 2000; Crespo & Nicol, 2006; Kazemi & Franke, 2004; Son & Crespo, 2009; Stacey et al., 2001) and have found that analyzing student work has produced positive outcomes in terms of knowledge development.

Research has also shown that a focus on student work analyses can also be powerful in changing teachers' beliefs (Philipp, Armstrong, & Bezuk, 1993; Philipp et al., 2007; Vacc & Bright, 1999). For example, Philipp et al. (2007) found that PSTs developed much more sophisticated beliefs about mathematical understanding and learning when they explored mathematical topics by way of analyzing students' work. So, in both knowledge and belief instances, analyzing student work has shown promise as a powerful intervention for helping PSTs prepare to teach mathematics effectively.

Table 1
Beliefs for Effective Mathematics Teaching

Category	Belief(s)
Beliefs about mathematics:	(1) Mathematics is a web of interrelated concepts and procedures (and school mathematics should be too).
Beliefs about learning or knowing mathematics:	(2) One's knowledge of how to apply mathematical procedures does not necessarily go with understanding the underlying concepts.
	(3) Understanding mathematical concepts is more powerful and more generative than remembering mathematical procedures.
	(4) If students learn mathematical concepts before they learn procedures, they are more likely to understand the procedures when they learn them. If they learn the procedures first, they are less likely ever to learn the concepts.
Beliefs About Children's (Students') Learning and Doing Mathematics	(5) Children can solve problems in novel ways before being taught how to solve such problems. Children in primary grades generally understand more mathematics and have more flexible solution strategies than adults expect.
	(6) The ways children think about mathematics are generally different from the ways adults would expect them to think about mathematics. For example, real-world contexts support children's initial thinking whereas symbols do not.
	(7) During interactions related to the learning of mathematics, the teacher should allow the children to do as much of the thinking as possible.

Extensions to the Current Literature Base

Despite these promising findings with both practicing and PSTs, there remains a shortage of studies that have specifically examined the simultaneous development of mathematical knowledge and beliefs in PSTs within the context of student work analysis. This study seeks to help fill this gap in the literature by focusing on PSTs and student work analysis designed to impact knowledge and belief development for effective mathematics teaching.

Furthermore, while student work has been shown to impact teachers' mathematical knowledge and beliefs, it has also been reported that recognizing the meaning of students' work and making sense of students' mathematical thinking are challenging tasks for PSTs (Son & Crespo, 2009). Therefore, it is important for PST preparation programs to not only include analysis opportunities but also to provide structured ways for PSTs to interact with them. Consequently, another purpose of this study was to create and empirically test structured student work analysis activities as they apply to affecting PSTs' MKT and related mathematical beliefs.

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The MKT framework encompasses, as was outlined earlier, a multitude of knowledge types. This study focused only on the CCK domain from the MKT framework because a current, valid instrument exists for measuring the levels of CCK. Currently, no instruments have been developed to measure specialized content knowledge (SCK) or mathematics on the horizon (the remaining two elements of SMK) in a reliable or valid manner. Furthermore, no quantitative measures have been developed for use with the elements of the PCK half of the knowledge framework. Undoubtedly, future research is needed to unpack and measure the remaining elements of MKT in order to be able to empirically test ways to development them.

Two research questions guided these purposes:

1. What is the impact of a structured analysis of student work on PSTs' CCK?

2. What is the impact of a structured analysis of student work on PSTs' beliefs about effective mathematics teaching?

These research questions were developed through inquiries from the researchers' own teaching experiences and work with MKT and beliefs. It was hypothesized that elements of MKT might be influenced through the use of student work analyses. Moreover, it was hypothesized that beliefs about effective mathematics teaching might be influenced through the analyses. Certainly, as outline earlier, the research literature supports these hypotheses based on previous work with practicing teachers and through separate knowledge and beliefs studies.

Treatment's Theoretical Framework

Sociocultural learning theory served as the framework and justification for the selection and setup of the treatment activities. This was done to ensure that PSTs could meaningfully complete a difficult task like analyzing student work. Sociocultural learning theory claims "learning, thinking, and knowing are relations among people in activity in, with, and arising from the socially and culturally structured world" (Lave, 1991). Vygotsky (1978) stated that learning is embedded within social events and social interaction plays a fundamental role in the improvement of learning. Furthermore, Rogoff (1994) described a sociocultural framework for learning that has had considerable impact on the conceptualization of this study. This theory, "transformation through participation," says learning takes place when people participate in shared endeavors. There is neither a sole focus on the learner nor the teacher, but rather a joint and collective effort. Involvement in social activities produces true learning.

Based on sociocultural theory, PSTs must participate as well as socially negotiate, discuss, and reflect during their preparation programs in order to learn in a meaningful fashion. Analyzing student work certainly provides fertile grounds for such vicarious, social opportunities while remaining situated in an authentic teaching context (e.g., Crespo, 2000; Crespo & Nicol, 2006; Kazemi & Franke, 2004; Philipp, Armstrong, & Bezuk, 1993; Philipp et al., 2007; Son & Crespo, 2009; Stacey et al., 2001; Vacc & Bright, 1999). However, to ensure this type of interaction with the student work, this study created and implemented a student work analyses protocol designed to engage PSTs in a social experience (both small and large group) as they learned mathematics content and teaching strategies. The specifics of this protocol are discussed next.

Research Methods

Participants and Context. This study explored PSTs' growth in CCK and beliefs about effective teaching through quantitative data analyses. A blended content and methods course at a major university in the southeastern United States served as the setting for both the control and treatment groups. The course focused simultaneously on mathematical content knowledge and introductory methods of teaching mathematics. The 42 participants in the study were randomly assigned to either the treatment (n=21) or the control group (n=21). All participants (2 male, 40 female) were undergraduate students enrolled in their first semester of a preparation program for prospective elementary school teachers. This program leads to the degrees of Bachelor of Arts in Education and Master of Education as well as recommendation for state teaching certification for grades K-6. Prior to entering the study, each participant had completed a mathematics course at or above the college algebra level, although it should be noted that some participants had taken additional mathematics courses.

Treatment. The treatment for this study was the participants' involvement in a student work analysis protocol (see Appendix A) developed by modifying an existing professional development protocol from the National School Reform Faculty website. The revisions made were to alter the protocol from one designed for knowledge development with practicing teachers to a protocol for university-level use with prospective teachers that focused on both mathematical knowledge and beliefs. The treatment protocol maintained the original suggestions about collecting student work.

Outside of the treatment protocol, the treatment and control groups were very similar. Both the treatment and control class meetings were video recorded and later annotated to document the mathematics topics covered and the modes of instruction used. The major difference in the groups' instruction was that the treatment group focused on student work as the catalyst for all mathematical content and belief discussions while the control group focused on direct instruction and group discussion to introduce mathematical topics. Many times for the control group, discussions about the seven beliefs did not arise from the lecture and open discussion. Both groups covered the same content (i.e., number and operation topics that included the base ten system, numbers in other bases, and addition, subtraction, multiplication, and division in base ten) at almost identical paces from the same textbook.

Specific Ties to Beliefs. The treatment protocol guided the discussion around each new mathematical topic that was introduced. Student work was carefully selected to represent various levels of students' mathematical understanding as well as address the seven beliefs as outlined by Philipps et al. (2007). For example, a video of a student struggling to use an algorithm but succeeding with a drawing was chosen to help address belief 6 (*the ways children think about mathematics are generally different from the ways adults would expect them to think about mathematics. For example, real-world contexts support children's initial thinking whereas symbols do not*).

Additionally, written work was chosen from a local elementary school to show both a student who completed a two-digit by two-digit multiplication problem correctly using a standard algorithm and a student that solved it incorrectly. Participants, per the protocol, analyzed for correctness, planned how they would proceed with the students given their current levels of understanding, and discussed what misconceptions may have caused the error – in this case the error was multiplying only the ones places together and tens places together leaving out the two other partial products. This written work addressed belief 2 (*one's knowledge of how to apply mathematical procedures does not necessarily go with understanding the underlying concepts*).

The remaining student work for the treatment group was selected in this manner and covered all seven of the beliefs similarly to what is shared here.

Specific Ties to Content Knowledge. When being introduced to a new topic, the treatment group first analyzed several pieces of student work in small groups that served as the basis of instruction in the course. This work was selected to highlight various levels of understanding of the given mathematical topic as well as many different solution strategies. These included examples of correct standard algorithm use, incorrect standard algorithm use, common misconceptions, invented strategies, and unusual strategies – both correct and incorrect. The goal was to provoke conversation about the mathematical content and force PSTs to draw on their own conceptual understanding.

To that end, PSTs were required to judge the students' level of understanding and plan the next steps for instruction based on that understanding. For example, participants might choose to suggest re-teaching an underlying concept that a student misunderstands, create a more challenging problem for a student who shows good understanding, or create a new problem to expose a potential misconception. Whatever the case, these discussions required them to address the mathematical operations and concepts presented to them as well as discuss what represented good and poor understanding. This stood in stark contrast to the control group where the instructor worked examples in class to introduce a new topic.

After the small group analysis, a whole group discussion was used to debrief the treatment group on everything that was discussed about a given piece of student work to ensure connections were made to the PSTs' mathematical content understanding.

Data Collection. Two data sources were utilized for this study. First, quantitative data came in part from an established mathematical CCK exam (Hill, Rowan, & Ball, 2005) (see Appendix B for an example question). The instrument has been piloted on a large scale through California's Mathematical Professional Development Institute (CMPDI) with reliability of 0.84 or higher for all forms. Hill, Rowan, and Ball (2005) reported that the items used for this instrument were subjected to a content validity check and contained adequate coverage across the number concepts, operations, and patterns. Furthermore, Hill, Ball, and Schilling (2004) reported that the items represented teaching-specific mathematical skills and could reliably discriminate among teachers and meet basic validity requirements for measuring teachers' CCK.

The CCK data were collected from a pretest (during week 1 of the study) and from a posttest (during week 8 of the study) using two validated, parallel forms. Both forms contained 22-24 questions that were scored as either correct or incorrect using a multiple-choice format. Some questions required participants to select all correct answers from the list of possible choices. In those cases, the question was only scored as correct if the participant selected all correct choices and no incorrect choices. Each participant was given a raw score and then a scaled score out of 100. The scaled score was used for analysis purposes.

Quantitative data also came from an existing 16-item survey instrument designed for the seven beliefs previously discussed (Philipp et al., 2007) (see Appendix C for an example question). The researchers were trained to score this survey using the practice modules provided by the survey developers. These modules contained a rubric, examples of rubric use, and multiple survey responses to be recorded for scorer calibration purposes. The researchers read through all rubrics and examples of rubric use, and then participated in the practice-scoring portion of the module. This process produced a reliability coefficient of 0.93 between the researchers and standards set forth and validated by the survey developers.

The ordinal data from the beliefs survey were collected from each participant through a pretest (week 1 of the study) and posttest (week 8 of the study) score for each of the seven beliefs discussed earlier. Since this instrument lacked parallel forms, the beliefs data were collected using the same form. These initial pre- and posttest integer scores ranged from 0 – 4 and were determined from the rubrics validated for the instrument.

Findings

To answer the first research question, an ANCOVA was used with the CCK data. The results revealed that no significant difference was present (p = 0.599) between the treatment and control groups when controlling for the pretest scores (Table 2).

Table 2

ANCOVA Results for the CCK Data Analysis

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	3922.480[a]	2	1961.240	12.546	.000	.392
Intercept	3818.596	1	3818.596	24.428	.000	.385
PRETEST_CCK	3700.622	1	3700.622	23.674	.000	.378
Group	44.030	1	44.030	.282	.599	.007
Error	6096.411	39	156.318			
Total	125255.939	42				
Corrected Total	10018.891	41				

Tests of Between-Subjects Effects
Dependent Variable: POSTTEST_CCK

As the mean scores show (Table 3), the growth was nearly the same in both groups. In fact, a dependent samples t-test revealed that both the control group (p=0.002) and the treatment group (p<0.001) made significant gains from pre- to posttest. While the student work analysis treatment did not produce significant gains beyond that of the control group, it is important to note that a wider focus on beliefs did not hinder the development of CCK in the treatment participants compared to the control group.

Table 3

CCK Mean Scores

Group	Pretest Score	Posttest Score
Treatment	42.2	54.6
Control	37.8	50.1

In order to answer the second research question, a Chi-Square analysis was used to examine the data from the beliefs survey. This analysis revealed a significant difference between the treatment and control groups for six of the seven beliefs (Table 4). Only belief 3 (understanding mathematical concepts is more powerful and more generative than remembering mathematical procedures), p=0.465, saw no significant difference between the treatment and control groups.

Table 4.
Belief Change Score Significance Values

Belief	Pearson Chi-Square Value	Degrees of Freedom	p-value
1	11.053	2	0.004
2	12.185	2	0.002
3	1.533	2	0.465
4	7.795	2	0.020
5	10.462	2	0.005
6	27.300	2	< 0.000
7	9.333	2	0.009

The cross-tabulation in Table 5 below allows for further interpretation beyond the p-value and significance. The actual counts reveal that the treatment group experienced larger positive changes in their beliefs about effective mathematics teaching. While only four treatment participants experienced no belief changes (compared to 12 for the control group), seven treatment participants experienced an increase of two or more belief levels (compared to none in the control group). This finding suggests a significant impact from the treatment activities. This lopsided pattern of larger numbers of participants with high change in the treatment group and larger numbers of no change in the control group was true for all seven of the beliefs measured except belief 3. The remaining six change score cross-tabulation tables are provided in Appendix D.

Table 5
Belief 1 Crosstabulation Values

Group	Change score 0, expected count	Change score 0, actual count	Change score 1, expected count	Change score 1, actual count	Change score 2, expected count	Change score 2, actual count
Treatment	8	4	9.5	10	3.5	7
Control	8	12	9.5	9	3.5	0

To ensure that the control group did not have a ceiling effect on their change scores, a Chi-Square analysis was also run on the pretest belief scores for all participants (Table 6). These results show that the pretest scores were not significantly different between the control and treatment groups for beliefs 1, 3, 4, 5, and 7. Furthermore, the treatment group had significantly higher pretest scores for belief 2 (Table 7). Only with belief 6 did the control group have a higher pretest score and thus a higher potential for a ceiling effect on their change scores (Table 8). Overall, the data show that treatment group participants experienced significant changes in beliefs (towards having beliefs consistent with effective mathematics teaching) beyond that of the control group.

Table 6
Beliefs Pretest Significance Values

Belief	Pearson Chi-Square Value	Degrees of Freedom	p-value
1	6.795	2	0.079
2	6.985	2	0.030
3	3.887	2	0.274
4	6.467	2	0.091
5	0.220	2	0.896
6	9.674	2	0.022
7	1.024	2	0.500

Table 7
Belief 2 Pretest Crosstabulation

Group	Score 0, expected count	Score 0, actual count	Score 1, expected count	Score 1, actual count	Score 2+, expected count	Score 2+, actual count
Treatment	16.5	13	4	7	0.5	1
Control	16.5	20	4	1	0.5	0

Table 8
Belief 6 Pretest Crosstabulation

Group	Score 0, expected count	Score 0, actual count	Score 1, expected count	Score 1, actual count	Score 2+, expected count	Score 2+, actual count
Treatment	8.5	13	8.5	5	4	3
Control	8.5	4	8.5	12	4	5

Discussion

To revisit, the two research questions guided this study:

1. What is the impact of a structured analysis of student work on PSTs' CCK?

2. What is the impact of a structured analysis of student work on PSTs' beliefs about effective mathematics teaching?

The results of the quantitative analysis showed that, first, the student work and thinking analysis protocol treatment had the same effect on the CCK of the treatment group as the control activities had on the group. It is important to recognize that the treatment group experienced gains in CCK equal to that experienced by the control group (both were statistically significant from pre- to posttest). There are various ways to interpret this result. It may have been in part because of the initial belief filters held by all participants. Beliefs have the ability to prevent PSTs' from garnering pertinent topics that require thinking in ways different from when they were K-12 students (Philipp et al., 2007). It is possible that this filtering prevented this study's treatment group's growth beyond that of the control group, which has been suggested as possible by Ambrose (2004). Although belief change was shown in the treatment group, initial belief filters were certainly possible. In any case, it is also important to note that the treatment group maintained a statistically similar significant growth as the control group. So, changing the structure of instruction to include student work analyses and a wider focus on beliefs did not hinder the growth of the treatment group.

The PSTs' beliefs about effective mathematics teaching, however, saw a significant change in the treatment group as compared to the control group through the Chi-Square analysis. Of the seven beliefs measured, six were significantly impacted by the treatment activities. This suggests that the treatment activities were overall able to create a context for developing mathematical beliefs in the university classroom without the need to be in a K-12 classroom. This supports the work of Philipp and others (2007) who found that university course settings could have impacts on teachers' beliefs about effective mathematics teaching – in some cases even more so than practical classroom settings.

These findings help to paint a picture of how student work can be used at the university level to impact knowledge and beliefs necessary for effective mathematics teaching (Hill, Ball, & Schilling, 2008; Philipp et al., 2007). Previous studies have shown that using student work in the university classroom can be productive in PST development for teaching, but this study helps to show ways in which knowledge and beliefs may be developed simultaneously as is necessary in mathematics (Ambrose, 2004).

Despite these positive findings, future iterations of research are still needed to continue to refine the role of student work and thinking in the preparation of PSTs. Although participants' content knowledge was not significantly impacted in the treatment group of this study, it is possible that the initial belief filters prevented knowledge growth. Future research can help to discover if knowledge gains may be latent until after belief changes have taken root, or if it may be necessary to refine the student work examples chosen for use. Additionally, perhaps context-rich student work coupled with verbalized student thinking may have provided more scaffolding for PSTs who were still grappling with their own belief changes. It is necessary to conduct future research to determine if different types of student work examples have different impacts on knowledge and belief growth. For example, would a focus on synchronous or asynchronous videos of students solving problems be more beneficial than written work?

Another reason to conduct future research is the limiting nature of some elements of this study. To begin, the small group sizes (n=21) may have affected the outcomes. The Chi-Square analysis, for instance, had very low expected values in some cells of the cross-tabulations. Furthermore, the length of the study could have caused two issues. Exposing the treatment group to the modified protocol for student work analyses for only eight weeks may have limited the knowledge growth if the initial belief filters prevented the accumulation of new CCK. Also, the brevity of the study could potentially lead to non-lasting effects on the PSTs' beliefs. A longer study with follow-up components would be needed to determine if the treatment activities are capable of creating lasting effects on PSTs' beliefs about the effective teaching and learning of mathematics.

There was also a possibility for a threat to the design validity of the study that could have affected the results. Specifically, there was a diffusion threat to the study's construct validity due to possible interactions between the control and treatment groups. Although these groups were not in any other courses together (they were members of separate cohorts at the university), they took classes in the same buildings. It is possible that they knew each other and had conversations about the classes they were taking.

Finally, the undergraduate and high school backgrounds of the participants varied. While all had credit for the required college algebra program prerequisite, some had more extensive mathematics backgrounds than others. The random assignment likely helped quell that potential issue, but it should be noted at a possible limiting factor of the study.

Even with the limitations outlined here, the findings suggest that using student work and thinking in the preparation of PSTs can have a positive impact on the development of the knowledge and beliefs necessary for effective mathematics teaching. The treatment protocol can be adapted to fit the needs of individual university classrooms, and the results suggest that it is a worthwhile undertaking for course instructors to provide student work analysis opportunities to prospective teachers. Although future iterations of research are certainly necessary to refine its role, this study provides evidence that student work and thinking analyses may play an important role in helping prospective teachers learn mathematics and teaching strategies in effective ways.

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APPENDIX A
USING STUDENT WORK PROTOCOL (ADAPTED FROM NSRF)

Selecting Student Work to Share
The selected student work will be used as the focal point of course lessons and in class discussions. The work itself will provide the mathematical topics as well as the teaching context for each lesson.

Choose student work that covers a variety of mathematical topics with a variety of solution types (i.e., traditional solution strategies, student invented algorithms, common errors, unique correct responses, etc.).

The key is to have enough artifacts and enough variety to drive the discussions and create situations that make PSTs examine their own understanding of the topic. Remember, student work comes in a variety of forms including videos (e.g., the Integrating Mathematics and Pedagogy [IMAP] project, Annenberg Learner website), written work collected from local schools, written work from PST education textbooks, etc.

Sharing and Discussing Student Work
Discussing student work requires a guide to help PSTs feel comfortable in sharing their thoughts about students' understanding as well as their own. Since learning is best accomplished through hands-on interactions, a structured dialogue format works well to promote thinking and learning about students' understanding and mathematical topics.

Ask the PSTs to assume that the students who completed the work or answered the questions in the videos were putting forth their best effort. Any mistakes or misconceptions are most likely honest.

Using the Protocol

Getting Started
The instructor should provide the student work example to the class and briefly introduce the mathematical topic of focus. If the example is written, the PSTs should have the opportunity to familiarize themselves with the work. If the example is a video, the instructor should play through the video two times to allow the PSTs to familiarize themselves with the work and the situation.

Small Group Session
Next, the following questions should be posed to the PSTs to discuss in small groups:

1. Is the solution correct? If not, what mistake is the student making? Explain your thinking.

2. Analyze the level of understanding the student has. What has the student done well? What concepts or understanding is the student lacking? Explain your thinking.

3. What should the next steps in instruction be for teaching this student? How would you expand their understanding of the concepts mastered and/or help them improve the missing concepts or understanding? Explain your thinking.

PSTs should be given 5 minutes at their small group (groups of three or four) setting to discuss their answers to the three questions. The instructor should ask the PSTs to read the questions and think about their responses for one minute. After that, each group member should take one minute to describe his or her thoughts to the group.

Reflecting on the Responses
After the small group discussions are complete, the instructor should bring the group back together as a whole. Debriefing should take place by posing the following questions to the whole group:

1. What is one thing that you learned while talking over the student work at your table? Why is this significant to you?
2. What new perspectives about the student, mathematical understanding, and/or mathematical content did your classmates provide you?

This discussion should be opened up to the entire group for volunteers to speak. If any major insights about the student work have been missed, the instructor should pose those questions and ask for ideas from the whole group.

The instructor should finish the protocol by providing a brief summary of the mathematical topic shown in the work example, the possible misunderstanding or exemplary understandings, and possible next steps for instruction.

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APPENDIX B
KNOWLEDGE INSTRUMENT SAMPLE

INSTRUCTIONS

- **Answer the questions by circling your choice, e.g.**

1. **During a unit on functions, Ms. Lopez asks her students to write journal entries on exponential growth. Which of the following journal entries illustrate exponential growth? (For each item below, circle EXPONENTIAL, NOT EXPONENTIAL, or I'M NOT SURE.)**

	Exponential	Not Exponential	I'm not sure
a) An example of exponential growth would be if you got 1% raise each year.	1	2	3
b) An example of exponential growth would be if a car increases in speed by 10 miles per hour ever second.	1	2	3
c) Exponential growth is when the y-axis increases faster than the x-axis. For example, if each time the x-coordinate goes up by 2, the y-coordinate goes up by 3.	1	2	3

APPENDIX C
BELIEFS INSTRUMENT SAMPLE

4. Here are two approaches that children used to solve the problem 635 – 482.

Lexi	Ariana
$\begin{array}{r} {}^{5}6{}^{1}3\ 5 \\ -\ \underline{4\ 8\ 2} \\ 1\ 5\ 3 \end{array}$ Lexi says, "First I subtracted 2 from 5 and got 3. Then I couldn't subtract 8 from 3, so I borrowed. I crossed out the 6, wrote a 5, then put a 1 next to the 3. Now it's 13 minus 8 is 5. And then 5 minus 4 is 1, so my answer is 153."	$\begin{array}{rrcr} 635 - & 400 & = & 235 \\ 235 - & 30 & = & 205 \\ 205 - & 50 & = & 155 \\ 155 - & \underline{\ \ 2} & = & 153 \\ & 482 & & \end{array}$ Ariana says, "First I subtracted 400 and got 235. Then I subtracted 30 and got 205, and I subtracted 50 more and got 155. I needed to subtract 2 more and ended up with 153."
4.1 Does Lexi's reasoning make sense to you? ○ Yes ○ No	**4.2 Does Ariana's reasoning make sense to you?** ○ Yes ○ No

a. Which child (Lexi or Ariana) shows the greater mathematical understanding? Why?

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APPENDIX D
CHANGE SCORE CROSSTABULATIONS

Table E-1. Belief 1 crosstabulation values.

Group	Change score 0, expected count	Change score 0, actual count	Change score 1, expected count	Change score 1, actual count	Change score 2, expected count	Change score 2, actual count
Treatment	8	4	9.5	10	3.5	7
Control	8	12	9.5	9	3.5	0

Table E-2. Belief 2 crosstabulation values.

Group	Change score 0, expected count	Change score 0, actual count	Change score 1, expected count	Change score 1, actual count	Change score 2, expected count	Change score 2, actual count
Treatment	10.5	5	6.5	9	4	7
Control	10.5	16	6.5	4	4	1

Table E-3. Belief 3 crosstabulation values.

Group	Change score 0, expected count	Change score 0, actual count	Change score 1, expected count	Change score 1, actual count	Change score 2, expected count	Change score 2, actual count
Treatment	10	8	6	7	10	6
Control	10	12	6	5	10	4

Table E-4. Belief 4 crosstabulation values.

Group	Change score 0, expected count	Change score 0, actual count	Change score 1, expected count	Change score 1, actual count	Change score 2, expected count	Change score 2, actual count
Treatment	13	9	6	8	2	4
Control	13	17	6	4	2	0

Table E-5. Belief 5 crosstabulation values.

Group	Change score 0, expected count	Change score 0, actual count	Change score 1, expected count	Change score 1, actual count	Change score 2, expected count	Change score 2, actual count
Treatment	13	9	4	4	4	8
Control	13	17	4	4	4	0

Table E-6. Belief 6 crosstabulation values.

Group	Change	Change	Change	Change	Change	Change

	score 0, expected count	score 0, actual count	score 1, expected count	score 1, actual count	score 2, expected count	score 2, actual count
Treatment	10	2	4	5	7	14
Control	10	18	4	3	7	0

Table E-7. Belief 7 crosstabulation values.

Group	Change score 0, expected count	Change score 0, actual count	Change score 1, expected count	Change score 1, actual count	Change score 2, expected count	Change score 2, actual count
Treatment	15	11	2.5	3	3.5	7
Control	15	19	2.5	2	3.5	0

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