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INFUSING PEDAGOGY IN MATHEMATICS CONTENT COURSES FOR FUTURE ELEMENTARY TEACHERS

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Abstract

To capitalize on limited time for mathematics teacher preparation, we suggest that mathematics content courses are fruitful venues for infusing elements of effective mathematics pedagogy. As content courses are typically taken before methods courses, doing so exposes pre-service teachers to mathematics teaching practices earlier in their teacher preparation programs. In this paper, we share examples of how we provide pre-service teachers the opportunity to build their mathematics content knowledge while concurrently building their pedagogical knowledge. We invite mathematics teacher educators to find ways to provide similar opportunities in their work with pre-service teachers.

Introduction

In many teacher preparation programs, pre-service teachers learn mathematics *content* in a separate setting from learning mathematics *pedagogy* (Steele and Hillen 2012). Content instruction and pedagogical instruction are often taught in different courses, separated both temporally and organizationally within teacher education systems. Yet "pedagogical knowledge is neither discrete nor conceptually separable from the knowledge of the mathematics content being taught" (p. 53). Like Steele and Hillen (2012), we wonder when and how teachers integrate their knowledge of content and pedagogy if we continue to teach them separately. In this paper, we suggest that mathematics content and mathematics pedagogy should, whenever possible, be taught concurrently. We offer mathematics content courses for pre-service elementary teachers (PSTs) as fruitful settings for infusing elements of effective mathematics pedagogy early in the teacher preparation trajectory. We describe how the Mathematics Teaching Practices put forth by the National Council of Teachers of Mathematics (NCTM) in *Principles to Actions* (NCTM 2014) undergird our decisions to infuse pedagogy within mathematics content courses and call on our mathematics teacher educator colleagues to find methods by which to do the same.

Though numerous textbooks exist (e.g., Beckmann 2018; Billstein, Libeskind, and Lott 2013; Van de Walle, Karp, and Bay-Williams, 2019) which are designed to increase PSTs' conceptual understanding of mathematics, we suggest that mathematics content courses can extend PSTs' learning to also involve problems of practice. That is, rather than only solving the typical mathematics tasks found in these textbooks, we can look for opportunities for PSTs to solve mathematics embedded in situations of everyday teaching. These types of situations include assessing students' mathematical work, using data to communicate to administrators and parents, designing classroom activities around particular mathematics concepts, etc. By including contexts such as these, PSTs are not only building their mathematics content knowledge, but also building their skills and confidence to enter the teaching profession.

Concurrently Learning Mathematics Content and Effective Pedagogy

Presently, many current and future teachers are expected to teach mathematics in ways that differ from how they experienced mathematics as learners (Koellner et al. 2007; Thanheiser and Jansen 2016). Koellner et al. (2007) caution that shifts to reform-based mathematics instruction will require “a great deal of learning on the part of teachers” (p. 275). To help facilitate this teacher learning, Thanheiser et al. (2010) posit a design principle for teacher preparation programs, namely that classes for PSTs should effectively model teaching for mathematical understanding. Thus, these programs should capitalize on multiple opportunities to expose PSTs to effective mathematics instruction. Though this exposure is likely to occur in mathematics *methods* courses (Amirshokoohi and Wisniewski 2018; Burton, Daane, and Giesen 2008; Steele and Hillen 2012), we suggest an additional opportunity is available in mathematics *content* courses.

One benefit of infusing pedagogy in mathematics content courses is that it introduces PSTs to effective mathematics instruction earlier in their teacher preparation trajectories. Content courses typically occur before methods courses in teacher preparation programs, allowing us to capitalize on limited time for mathematics teacher preparation. In *Building Support for Scholarly Practices in Mathematics Methods* (2017) – a volume containing contributions from over 40 mathematics teacher educators on theoretical approaches to teaching mathematics methods – contributing authors Harper, Sanchez, and Herbel-Eisenmann (2017) ask, “What can teacher education programs do to at least partially resolve the problem of time?” (p. 40). Their first suggestion is to improve instruction in mathematics content courses. They claim, “If we are going to change prospective teachers’ understanding and image of what it means to teach mathematics conceptually, then we have to give them the opportunity to learn mathematics in compatible ways” (p. 40). They suggest that infusing effective mathematics pedagogy in content courses will improve content knowledge, establish an early vision of effective mathematics instruction, and provide PSTs with a solid foundation to do advanced work in mathematics methods courses.

By linking mathematics content to effective mathematics pedagogy early in the teacher preparation trajectory, we may be helping PSTs to shift their identities as future teachers of mathematics. We are providing an earlier opportunity to “break the cycle” of mathematics anxiety among elementary teachers and positively influence PSTs’ pedagogical content knowledge, attitude, beliefs, and self-efficacy (Amirshokoohi and Wisniewski 2018). Rather than continuing their identities as mathematics learners from their K-12 experience, our mathematics content courses provide PSTs with opportunities to explore mathematics content through the lens of teaching. We have referred to this as having PSTs take off their ‘student hat’ and put on their ‘teacher hat’ (DiNapoli and Marzocchi 2017). Battey and Franke (2008) provided two case studies of teachers’ implementation of professional development and found that changes which occurred in the classroom were related to teacher identity and situation within communities of practice. They describe teaching as “a process of becoming a member in a defined group of practitioners with specific skills” (p. 128). Within the setting of a mathematics content course, our PSTs engage in tasks around the practice of teaching, providing opportunities to see themselves as members of the teaching community.

Others have infused mathematics content and pedagogy in settings such as a problem solving-focused professional development program (Koellner et al. 2007), a content-focused mathematics methods course around functions (Steele and Hillen 2012), and an elementary mathematics methods course (Amirshokoohi and Wisniewski 2018; Burton, Daane, and Giesen 2008). In their problem solving-focused professional development program, Koellner et al.’s

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(2007) participants were expected to draw on their specialized content knowledge to give explanations, make use of representations, evaluate ideas, identify misconceptions, make connections, and use language explicitly. PSTs in the content-focused mathematics methods course described by Steele and Hillen (2012) were given opportunities to move back and forth between positions of learner and teacher while analyzing student work, watching video, and considering multiple perspectives. Within the mathematics methods course described by Amirshokoohi and Wisniewski (2018), PSTs learned effective teaching strategies through content-focused tasks structured in a student-centered lesson plan model.

The work described above primarily focuses on infusing pedagogy within mathematics *methods* courses. As mentioned, we believe that exposure to effective mathematics pedagogy can occur earlier in the teacher education trajectory, within the context of mathematics *content* courses for future elementary teachers. We extend the work of our colleagues in mathematics methods courses (Amirshokoohi and Wisniewski 2018; Burton, Daane, and Giesen 2008; Steele and Hillen 2012) by providing classroom examples of infusing of pedagogy within a content course guided by the Mathematics Teaching Practices put forth by NCTM in *Principles to Actions* (NCTM 2014).

Context

Our work is situated in the context of two different public universities on each coast of the United States. In both universities, the mathematics content courses in which we aim to infuse elements of effective mathematics pedagogy are intended for PSTs seeking certification to teach early childhood and elementary school (P-3, K-6). As undergraduate students, PSTs complete a sequence of two mathematics content courses prior to enrolling in mathematics methods courses or participating in any field experiences. These mathematics content courses are designed similarly and taught by both authors, separately, at our respective universities. At the east coast university, the first course focuses on counting and cardinality, number and operations, and operations and algebraic thinking while the second course focuses on number and operations, ratios and proportional reasoning, geometry, and measurement. At the west coast university, the first course focuses on number and operations while the second course focuses on geometry, probability, and statistics.

Throughout these two courses, PSTs are given opportunities to explore mathematics content as participants in classroom activities that model and make explicit aspects of effective standards-based mathematics instruction. Course instruction is student-centered and inquiry-based, and provides opportunities to individually and collaboratively grapple with mathematical ideas, communicate and share ideas with peers, and interact with different representations of mathematical objects, especially representations that are concrete and visual. In these ways, our mathematics content courses are designed to leverage individual and collective engagement with challenging mathematics to develop conceptual understanding (Thanheiser and Jansen 2016). The course design emphasizes a social constructivist approach to learning by offering PSTs opportunities to make personal meaning of mathematical content and pedagogy through independent exploration, as well as through interaction and communication with peers and the instructor. Though the entire course is designed to enact elements of effective mathematics pedagogy, in what follows we provide specific examples of activities that engage PSTs in the act of deliberate problem solving as teachers, as guided by *Principles to Actions* (NCTM 2014).

Examples of Infusing Pedagogy

NCTM's *Principles to Actions* (NCTM 2014) recommends actions for education stakeholders to ensure high quality mathematics education. Contained in the document are eight strongly recommended and research-informed Mathematics Teaching Practices (see Figure 1) to consistently implement in every mathematics lesson. To provide readers with a sense of how we infuse mathematics pedagogy within a content course, we focus on two Mathematics Teaching Practices and provide a classroom example for each:

1. Support productive struggle in learning mathematics
2. Elicit and use evidence of student thinking

The provided examples serve to demonstrate the ways in which we strive to enact mathematics teaching practices in our own courses. With the motivation to infuse pedagogy in mathematics content courses, we encourage mathematics teacher educators to continue to find ways to do the same.

Mathematics Teaching Practices
Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

Figure 1. Eight Mathematics Teaching Practices from *Principles to Actions* (NCTM).

Support productive struggle in learning mathematics

NCTM (2014) suggests providing opportunities for, and supporting student engagement with, productive struggle in the context of learning mathematics. Although PSTs in our content courses routinely engage with tasks that elicit productive struggle, we also seek to help PSTs learn about the nature of productive struggle: what it looks like, what it's about, and why it's productive. Akin to Thanheiser and Jansen's (2016) efforts to help PSTs share their exploratory mathematical thinking, we consistently provide PSTs opportunities to describe how they are struggling, the mathematical idea(s) that constitutes the object of their struggle, and how this struggle supports their learning. By making productive struggle explicit in their own learning, we hope the PSTs will recognize, support, and nurture productive struggle in their future students.

An example of this occurred in a division of fractions task titled *What Remains?*, in which PSTs needed to interpret the meaning of a remainder for the first time. The *What Remains?* task was inspired by Beckmann's (2018) *What to Do With the Remainder?* class activity, located in the Division and Fractions and Division with Remainder lesson, within the Division unit. Although Beckmann's (2018) activity promotes conceptual understanding of fraction division, we extend this by asking pairs of PSTs to work collaboratively to solve a story problem both conceptually and procedurally and to additionally participate in an online discussion forum. The forum prompted PSTs to reflect on the ways in which they may have struggled with the problem. Sample PSTs' work accompanied by excerpts of their reflections can be seen in Figures 2 and 3, respectively.

The *What Remains?* task and reflection exercise was beneficial for PSTs because they could make clear connections between their problem-solving struggles and their mathematics learning (NCTM 2014). As illustrated in Figures 2 and 3, most PSTs encountered struggles reconciling the leftover 2/4 yards in the context of the problem, especially when juxtaposed with the procedural answer of 5 2/3 badges. This pair of students specifically struggled with interpreting the 'leftover' pieces in their conceptual solution as a portion of one badge. They conceptually arrived at an incorrect answer of 5 2/4 badges, and spent productive time thinking about why their conceptual and procedural answers were different, eventually relying on the repeated subtraction meaning of division to help them make meaning of the 'leftover' 2/4 yards. PSTs could read their peers' articulation of their similar productive struggles with this problem in the discussion forum, which helped them realize that including opportunities for struggle is a normal and necessary pedagogical practice for all mathematics teachers.

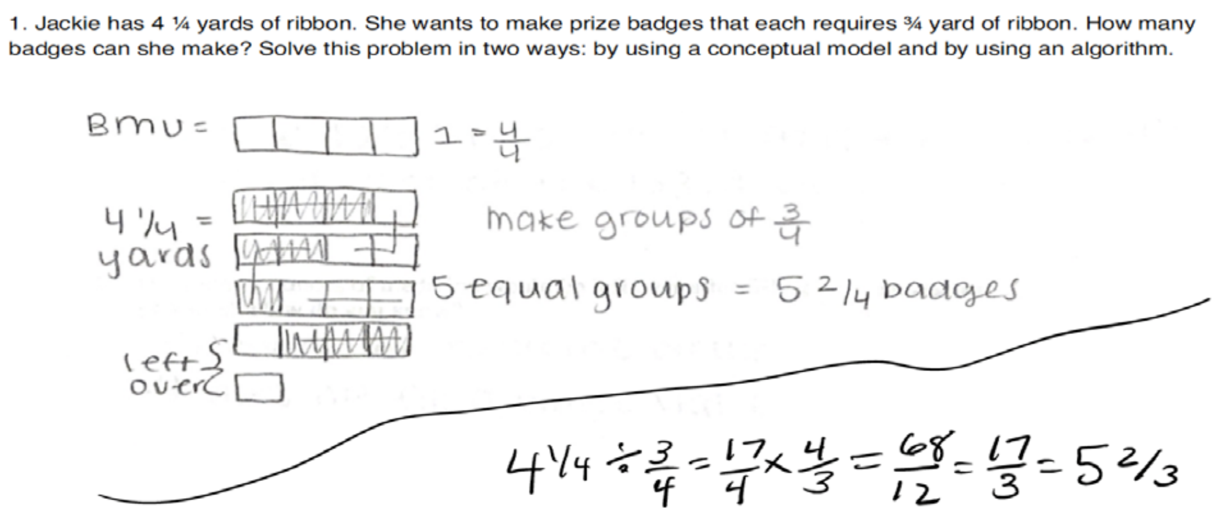

Figure 2. Sample PSTs' work on the *What Remains?* division of fractions task

1. How would you describe the ways in which you were struggling with this problem?
We struggled because we got different answers from the drawing and the algorithm. We talked about how they were different and talked about which one we thought was right. We even did the problem again on scrap paper.
2. What was the specific idea with which you were struggling? Be as specific as possible.
The 2/4 left over. We knew we had 5 whole badges but we were struggling with the 2/4 wondering why it wasn't 2/3.
3. How was this struggle productive for you, if at all?
It helped us see that we forgot to write the answer like numbers of badges. The 2/4 was about yards and we had to write it as a part of a badge because that's what division means. Seeing the 2/3 made us think how the 2/4 could be 2/3, and we remembered 3/4 yards makes a badge so 2/4 yards is only 2/3 of a badge because you have 2 out of the 3 quarter-yards you need.

Figure 3. Sample PSTs' productive struggle reflection for the *What Remains?* task

As instructors, we routinely use these productive struggle reflections as catalysts for discussions about the mathematical ideas that were not immediately apparent to PSTs – such as interpreting a remainder as a portion of the divisor – and how spending effort to wrestle with those ideas can pay dividends in meaningful learning. In these ways, PSTs' engagement in these learning opportunities helps integrate their knowledge of mathematics content and the effective pedagogy that helps develop it (Steele and Hillen 2012). In our content courses, PSTs see moments of struggle as learning opportunities which guide our instruction, a practice we hope they will enact in their future classrooms.

Elicit and use evidence of student thinking

According to NCTM (2014), effective mathematics teaching involves using evidence of student thinking to guide instruction. Busi and Jacobbe (2018), who quantitatively investigated the benefits for PSTs in courses that used student work samples as compared to courses that did not, found positive shifts in beliefs about how mathematics should be taught. Though we frequently make use of student data ourselves – by responding and adapting to PSTs' current understandings of the math content – we additionally strive to provide PSTs with the opportunity to enact this practice. Like other mathematics education instructors (e.g. Steele and Hillen 2012), we provide our PSTs with tasks and assessment items that involve interpreting and critiquing student work.

As an example, PSTs were given a summative group assessment on probability called *Probability Quiz*. The *Probability Quiz* assessment was inspired by Beckmann's (2018) *How Many Keys Are There?* class activity, located in the Counting the Number of Outcomes lesson, within the Probability unit. We extend these types of probability tasks by presenting PSTs with a fictional elementary student's probability quiz. PSTs were asked to assess the student's mathematical understanding of probability. Figure 4 shows an item from the fictional student's quiz, on which the fictional student is asked to determine the number of possible seating arrangements in a classroom with 16 desks and 16 students, and to find the probability of a student being seated in the front row. The fictional solution was intentionally designed to display a variety of understandings and misconceptions around the content. Throughout the semester, a

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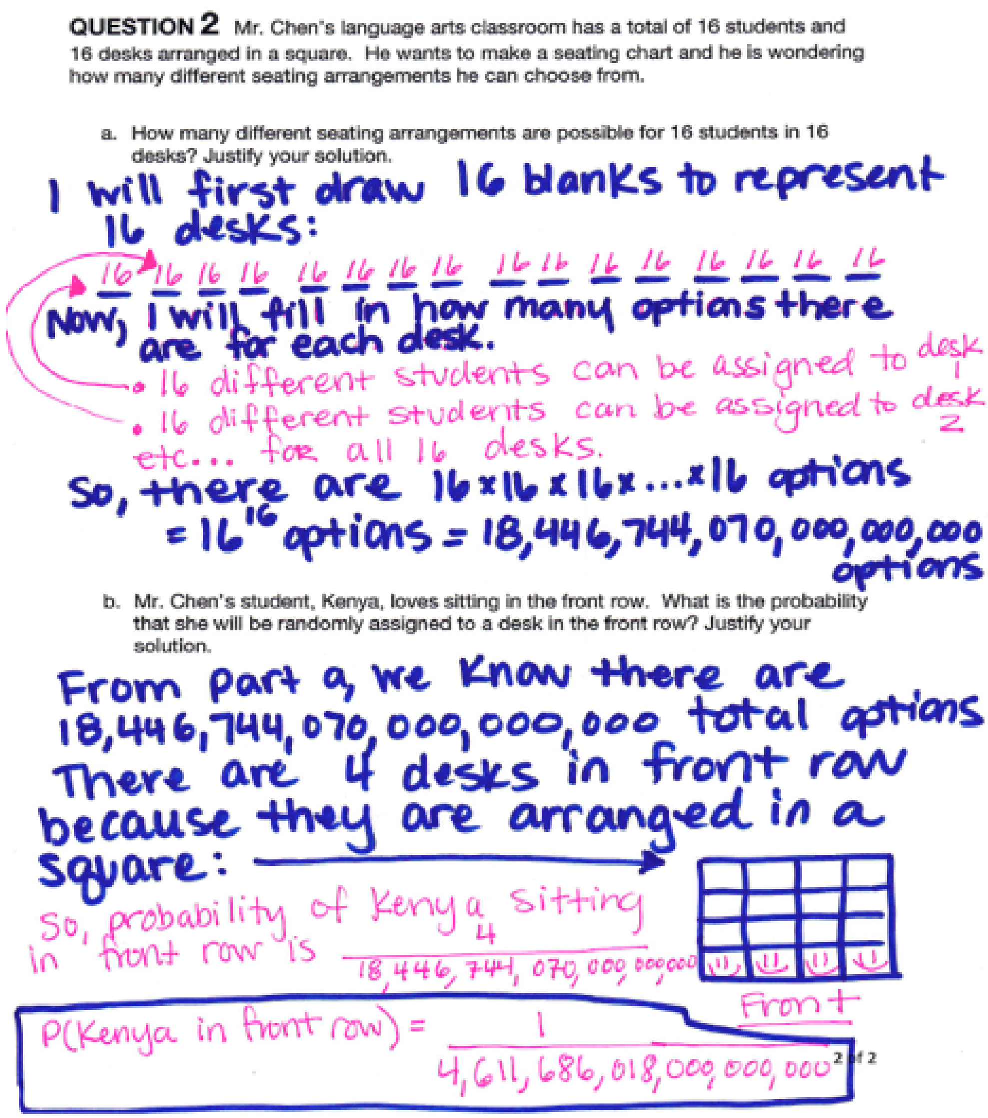
norm is established that mathematics teachers must seek to find understanding within incorrect student solutions. PSTs are made aware that just because a student's answer is wrong does not mean the student understands nothing about the concept, and that effective pedagogy is finding evidence of understanding and building from there. Figure 5 shows a sample group PST assessment of what the fictional student might understand or misunderstand around the content. This group of PSTs correctly identified the fictional student's understandings in how to set up the problems but misunderstandings in which numbers to use.

A task like *Probability Quiz* not only serves to build PSTs' own mathematics content knowledge but also provides them with the opportunity to enact NCTM's (2014) recommendation of assessing students' mathematical understanding. As mathematics teacher educators, we are able to enact this practice ourselves on a regular basis. Our work in infusing pedagogy in mathematics content courses extends this practice to our PSTs by providing them an early opportunity to practice the complex skill of interpreting student work.


QUESTION 2 Mr. Chen's language arts classroom has a total of 16 students and 16 desks arranged in a square. He wants to make a seating chart and he is wondering how many different seating arrangements he can choose from.

a. How many different seating arrangements are possible for 16 students in 16 desks? Justify your solution.

I will first draw 16 blanks to represent 16 desks:

16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16

Now, I will fill in how many options there are for each desk.

• 16 different students can be assigned to desk 1
• 16 different students can be assigned to desk 2
etc... for all 16 desks.

So, there are 16×16×16×...×16 options = 16^{16} options = 18,446,744,070,000,000,000 options

b. Mr. Chen's student, Kenya, loves sitting in the front row. What is the probability that she will be randomly assigned to a desk in the front row? Justify your solution.

From part a, we know there are 18,446,744,070,000,000,000 total options. There are 4 desks in front row because they are arranged in a square:

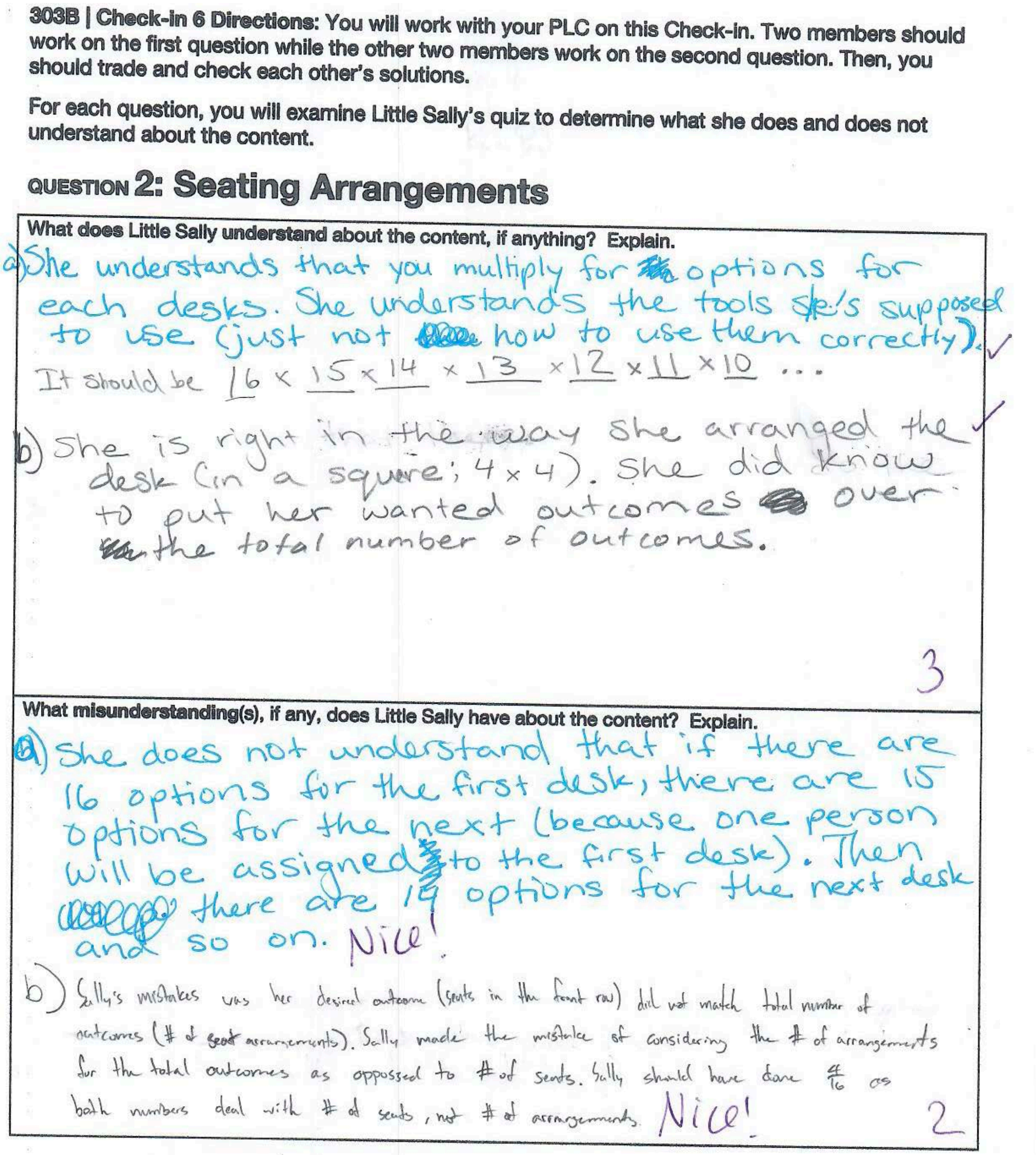
So, probability of Kenya sitting in front row is $\frac{4}{18,446,744,070,000,000,000}$

Front

P(Kenya in front row) = $\frac{1}{4,611,686,018,000,000,000}$

2 of 2

Figure 4. An item from *Probability Quiz*: a fictional student's probability quiz.



303B | Check-in 6 Directions: You will work with your PLC on this Check-in. Two members should work on the first question while the other two members work on the second question. Then, you should trade and check each other's solutions.

For each question, you will examine Little Sally's quiz to determine what she does and does not understand about the content.

QUESTION **2: Seating Arrangements**

What does Little Sally understand about the content, if anything? Explain.

a) She understands that you multiply for options for each desks. She understands the tools she's supposed to use (just not how to use them correctly). ✓

It should be 16 × 15 × 14 × 13 × 12 × 11 × 10 ...

b) She is right in the way she arranged the desk (in a square; 4 × 4). She did know to put her wanted outcomes over the total number of outcomes. ✓

3

What misunderstanding(s), if any, does Little Sally have about the content? Explain.

a) She does not understand that if there are 16 options for the first desk, there are 15 options for the next (because one person will be assigned to the first desk). Then there are 14 options for the next desk and so on. Nice!

b) Sally's mistakes was her desired outcome (seats in the front row) did not match total number of outcomes (# of arrangements). Sally made the mistake of considering the # of arrangements for the total outcomes as opposed to # of seats. Sally should have done 4/16 as both numbers deal with # of seats, not # of arrangements. Nice!

2

Figure 5. A sample pre-service teacher assessment of a fictional student's probability quiz.

Conclusion

Our work suggests that there are opportunities to infuse elements of effective mathematics pedagogy within mathematics content courses for future elementary teachers that extends beyond problems included in mathematics content textbooks. Above we shared select examples of what this looks like in our classrooms. We encourage mathematics teacher educators to be mindful of continued opportunities to infuse pedagogy in mathematics content courses. Doing so extends the work of researchers who have infused pedagogy within methods courses (Amirshokoohi and

Wisniewski 2018; Burton, Daane, and Giesen 2008; Steele and Hillen 2012). By infusing pedagogy in content courses, we are providing PSTs with additional and earlier opportunities to engage in the Mathematics Teaching Practices put forth by NCTM (2014). Infusing mathematics content and pedagogy within the same course is an improvement from typical models (wherein they are taught separately), as the process of learning to teach mathematics is “less additive (e.g., learn the content, then learn to teach it) and more iterative” (Steele and Hillen 2012, p. 53-54). An added benefit lies in providing PSTs with an earlier opportunity to shift their identities from that of a student to that of a teacher. By exposing PSTs to effective mathematics instruction earlier and more frequently, we hope they will ultimately implement effective planning and instruction in their future classrooms, thus, benefitting all students (Amirshokoohi and Wisniewski 2018; Thanheiser et al. 2010).

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