Chapter 3
Elementary Number Theory

The expression lcm(m,n) stands for the least integer which is a multiple of both the integers m and n. The expression gcd(m,n) stands for the biggest integer that divides both m and n.

Exercise. Find lcm and gcd in the TI-85 CATALOG and place them into your custom catalog.

Following the procedures of the first two chapters, compute each of the following integers.

	B&P	SGC	CAS
1. gcd(6,8)	____________	____________	______________
2. lcm(6,8)	____________	____________	______________
3. gcd(6,8)lcm(6,8)	____________	____________	______________
4. gcd(140,429)	____________	____________	______________
5. lcm(140,429)	____________	____________	______________
6. #4 * #5	____________	____________	______________
7. 140*429	____________	____________	______________

Exercise. Experiment with several different pairs of integers m and n computing gcd(m,n)*lcm(m,n) and m*n. What relationship is suggested by your experiment?

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I think it’s time to forget the pencil computations and rely solely on the technology **and our brains**. Don’t you agree?

SGC CAS

8. gcd(101+45,36^2)

______________________ _________________________

9. lcm(5(1+4^3),2^5)

______________________ ________________________

10. 36/gcd(36,16)

______________________ __________________________

11. lcm(38,100)/gcd(38,100)

______________________ __________________________

13. lcm(1524,5587)

______________________ __________________________

14. lcm(1524,5588)

______________________ __________________________

Notice that #13 and #14 show us that bigger integers don't necessarily yield bigger lcm's.

15. gcd(10^13,1050)

______________________ _________________________

16. lcm(14^6,15^10)

______________________ _________________________

17. gcd(your ss#,a friends' ss#)

______________________ _________________________

18. lcm(your ss#,same friends' ss#)

______________________ _________________________

Two integers are said to be <u>relatively prime</u> if the biggest integer that divides both of them is 1.

Exercise. List all pairs of integers from 100 to 105 that are relatively prime.______________________________

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An integer is said to be <u>prime</u> if its only divisors are 1 and itself. For example 2,3,5,&7 are the only primes less than 10. What are the primes between 10 and 20?___

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The MAPLE command "isprime" determines whether or not a given integer is prime. Use it to see if the following numbers are prime: 29_____, 35______, 1537_______, your ss#______,10!+1_______, 329891_______.

The **FUNDAMENTAL THEOREM OF ARITHMETIC** states that every integer can be factored in a unique way into a product of powers of primes.

Exercise. Use paper and pencil, aided by the TI-85 if necessary, to write each of the following numbers as a product of powers of primes:

i. 10!= ____

ii. 340704= ____________________________________

The MAPLE command "ifactor" does this automatically. Now use it to compute the prime factorization for each of I & ii above as well as for

iii. 10!+1= ____________________________________

and iv. your ss# = ______________________________

(Notice how just adding 1 to 10! reduced the number of prime factors.)

For two integers m & n with $n \geq m$ we say the <u>remainder of n divided by m is r</u> if n = qm + r, for integers q &r with $0 \leq r < m$. In MAPLE this remainder is denoted by "n mod m". In the TI-85 CATALOG it is denoted by "mod(n,m)".

Exercise. With paper and pencil compute the following remainders:

24 mod 5 =__________ 156 mod 7 =_____________

32 mod 2 =__________ 57 mod 2 =______________

Now use the appropriate commands to verify these results with both the TI-85 and MAPLE.

Compute each of the following remainders using the TI-85 first and MAPLE next, as usual.

19. 13200 mod 134

____________________ ______________________

20. (12^3+53) mod 26

21. 400*23 mod 19

22. 1321 mod (5*7)

23. 45^16 mod 4

Exercise. Which answer in number 23 do you think is correct?__________________. Explain______________________

Exercise. Explain why an integer n is even exactly when n mod 2 =0, and n is odd exactly when n mod 2 =1.

Recall the statement of the **BINOMIAL THEOREM.**

(m + n)^k =

Exercise. Use the binomial theorem to explicitly expand, (ie. compute the binomial coefficients)

i. (x + y)^3 =

and

ii. (n +1)^4=

Exercise. Explain why every power of an odd integer must be an odd integer also.______________________________

You should be able to compute each of the following **using brain only**. Do so and then check your answer using both technologies. Record your results in the appropriate places.

	Brain	SGC	CAS
24. 1342 mod 2	___________	___________	___________
25. (10!+1) mod 2	___________	___________	___________

26. 27^7 mod 2 ____________ ____________ ____________

27. 27^9 mod 2 ____________ ____________ ____________

28. 27^11 mod 2 ____________ ____________ ____________

Exercise. Explain why the TI-85 thinks 27^11 is even, when we, and MAPLE, know better.

Reflection: 1. What are the main things you learned from Chapter 3?

2. What's the hardest thing to understand from Chapter 3?