At the end of Chapter 4 we learned Newton’s method for searching for roots of a function $y = f(x)$; that is, find values of $x$ so that $y = 0$. Basically the idea is to see where successive tangent lines cross the x-axis. If we start with some value of $x$, henceforth referred to as an answer, we get the next value of $x$, the new answer, by the following formula:

$$\text{new answer} = \text{answer} - \frac{f(\text{answer})}{f'(\text{answer})}.$$  

Consider the example $f(x) = x^2 - 3$. So $f'(x) = 2x$ and Newton’s formula becomes

$$\text{new answer} = \text{answer} - \frac{(\text{answer})^2 - 3}{2 \text{answer}}.$$  

If the initial answer is 1, what is the new answer? Now let this number be the answer, what is the next new answer? To continue this process to find the next new answer type the following command onto the screen (Ans is gotten from the keyboard via 2nd ANS):

$$\text{Ans} - \frac{(\text{Ans}^2 - 3)}{(2 \text{Ans})}.$$  

Pressing ENTER then will activate Newton’s method on the previous answer. What do you get? Continue pressing ENTER until the numbers stabilize to 10 decimal places. What is the answer? How does this compare to $\sqrt{3}$?

Perhaps it is time to write a simple program for doing Newton’s method on our machine. Choose PRGM from the keyboard and EDIT from the screen menu. We are asked for the program’s name. Notice the TI is already in the alpha mode, so all we need to do is type in the letters. Let’s name our program NEW. Press ENTER and then type in the following program:

$$\text{Ans} - \frac{\text{evalF}(\text{y1, x, Ans})}{\text{evalF}(\text{der1( y1, x) , x, Ans})}.$$  

pressing ENTER ends the editing session and stores the program called NEW. (Both evalF and der1 can be found in the general CATALOG or in the special calculus package accessed by choosing 2nd CALC from the keyboard.)

This program will apply Newton’s method to whatever function you have defined as y1 in the graph package, starting at whatever answer you have put into the machine (right number on screen). To apply it on our above example define $y_1 = x^2 - 3$ in the graph package. Exit to the home screen, press 1 and ENTER. This makes 1 the previous answer (ANS). Press PRGM and choose NAMES from the screen menu. Choose NEW and press ENTER. Pressing ENTER now causes our program to act on each previous answer. In a few iterations you should be able to obtain $\sqrt{3}$ to 10 decimal places. Try it. Try the process with the initial answer = 0. (Press 0, then ENTER, then invoke our program NEW. What happens? Why?)

Next try starting at the initial answer -1. What does Newton’s method yield to 10 decimal places?

Use this program on the homework problems you did on page 191 and compare the results with your previous calculations.