In Section 1 of Chapter 3 we learned that the derivative of a function \( f \) at the input value \( x \) is the limiting value of the slopes of the chords drawn from \((x, f(x))\) to nearby points on the graph. This slope from \((x, f(x))\) to the nearby point \((x + h, f(x + h))\) is the difference quotient

\[
DQ: \frac{(f(x + h) - f(x))}{h}
\]

for small positive and negative values of \( h \). And its’ limiting value we defined to be the slope of the tangent line to the graph at \((x, f(x))\).

Also in Section 1, we saw how to find a formula for this derivative in terms of the value \( x \) for some very simple functions. Here we will try to conjecture what such a derivative formula might be for a much more sophisticated function; namely, we’ll consider the function \( f(x) = \ln(x) \).

We’ll investigate the limit of the difference quotient at the three values \( x = 1, 2, \) & 3.

1. Compute each of the difference quotients for \( f(x) = \ln(x) \) directly:

   - for \( x = 1 \) & \( h = 10^{-10} \) 
     \[ DQ = \text{________________________} \]
   - for \( x = 1 \) & \( h = -10^{-10} \) 
     \[ DQ = \text{________________________} \]
   - for \( x = 2 \) & \( h = 10^{-10} \) 
     \[ DQ = \text{________________________} \]
   - for \( x = 2 \) & \( h = -10^{-10} \) 
     \[ DQ = \text{________________________} \]
   - for \( x = 3 \) & \( h = 10^{-10} \) 
     \[ DQ = \text{________________________} \]
   - for \( x = 3 \) & \( h = -10^{-10} \) 
     \[ DQ = \text{________________________} \]

2. Sketch the graph of each DQ for each value of \( x = 1, 2, \) & 3 and use ZOOM and TRACE to estimate the value near 0. (The variable in each DQ is \( h \), but the TI-85 recognizes only \( x \) as an independent variable. So we have to replace \( h \) in the above DQ with \( x \). This shouldn’t be too confusing since the \( x \) in the above DQ is to be replaced with the numbers 1, 2, & 3 anyway.)

So the graph of DQ corresponding to the point \((1, \ln(1))\) is the graph of

\[
y = \left( \ln(1 + x) - \ln(1) \right) / x \quad \text{and} \quad \text{limit as } x \to 0 \approx \text{________________________}
\]

At the point \((2, \ln(2))\) the graph is that of

\[
y = \left( \ln(2 + x) - \ln(2) \right) / x \quad \text{limit as } x \to 0 \approx \text{________________________}
\]

At the point \((3, \ln(3))\) the graph is that of

\[
y = \left( \ln(3 + x) - \ln(3) \right) / x \quad \text{limit as } x \to 0 \approx \text{________________________}
\]

3. Finally, simply sketch the graph of \( y = \ln(x) \) in the ZDECM window. Choose MATH from the screen menu, then choose dy/dx from the MATH screen menu. TRACE is automatically activated. Move the cursor to \( x = 1 \) and press ENTER. Do the same for each of \( x = 2 \) and \( x = 3 \). We obtain dy/dx = _______ at \( x = 1 \), dy/dx = _______ at \( x = 2 \), and dy/dx = _______ at \( x = 3 \).

What appears to be the relationship between the value of \( x \) and that of dy/dx for \( y = \ln(x) \)? _______